Pupil Replication
for exo-planet imaging

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Overview

- Principle
- Analysis: 1-D, 2-D, errors
- Optics design
- Simulation of applications

Slides:
- (3)
- (4)
- (1)
- (7)
Star flux problem

- Flux from host star dominates exoplanet
- Problems:
  - contrast
  - inner working angle
- Techniques to suppress scattered host flux:
  - coronography
  - pupil apodisation
  - Interferometry
- Pupil replication helps

Diffraction limited PSF, x-axis in $\lambda/d$
Principle

- Start with normal telescope
- Include replication optics and second imaging lens.
- Axial wavefront is continuous (blue), like larger telescope.
- Non-axial wavefront is discontinuous (red).

from ApJL 618
Expectations

- Consider 3-fold replication in one dimension
- *Axial* pupil wavefront is 3x wider and 3x fainter
- *Image* of unresolved axial star is 3x *narrower* and 3x *brighter*
- Sidelobes are 3x *fainter*
- Image stop to remove star flux can be smaller
- *Inner Working Angle can be improved*
Analysis (one dimension)

in 1 dimension, plane wave on axis, hard edged pupil:

wave + pupil:  
\[ \begin{array}{c}
\text{wave} + \text{pupil}:
\end{array} \]

imaged:  
\[ \begin{array}{c}
sin(x)/x = \text{sinc}(x)
\end{array} \]

\[ \begin{array}{c}
sinc(x) (1 + 2\cos(x))
\end{array} \]

\[ T = A \left( 1 + 2 \cos \left( 2d \frac{\pi x}{\lambda} \right) \right) \text{sinc} \left( d \pi \left( x - \sin(\alpha) \right) / \lambda \right) \]
Two Dimensions

- Square pupil (seamless replication) then: 2-D amplitude $T$

$$T = T_x T_y$$

- $T$ for two replications 1 and 2
  - 1 is on axis, 2 is adjacent

$$T = T_{x1} T_{y1} + T_{x2} T_{y2}$$

- errors imposed on 2
Including Errors

- Pupil 1: $T_{1x}$ and $T_{1y}$ are sinc functions
- Pupil 2 has errors in x (and y) direction:
  - shift $s$, shear $h$, piston $p$, tilt $f$ and tip $g$

\[
T_{2x} = A_2 e^{i2\pi\left(p + \frac{x}{\lambda}(d+s)\right)} \text{sinc}\left(d\pi\left(\frac{x - \sin(\alpha)}{\lambda} - f\right)\right)
\]

\[
T_{y2}(y) = A_2 e^{i2\pi\left(p + \frac{y}{\lambda}(h)\right)} \text{sinc}\left(d\pi\left(\frac{y - \sin(\alpha)}{\lambda} - g\right)\right)
\]
Error Evaluation

- 2-D analyses; horizontal crosssection
- If PSF(no error) - PSF(with error) < 10^{-10}
  - example: shift = d 10^{-3} (10^{-5}) => 100 (1) micron for 1 cm pupil
  - difference images =>
- Alternative method: PSF of (error – no error)
Replication Optics (2-fold)

- Use 1 beamsplitter and mirrors
- Equal:
  - # reflections in each arm
  - optical path length
- Adjust last prism to vary replica separation => shift error avoided
- Can be cascaded
- Can be monolithic
Application

- Pupil replication applied to pupil *apodised* coronagraph
- Used simple not optimised function for attenuation $B$:
  - Super Gaussian
  - $x =$ off-axis angle
  - $c$ is adjusted to attenuate by $10^{-4}$ at pupil edge (both un- and replicated)
- Expect to
  - broaden the image core
  - reduce side-lobes

\[
B(x) = e^{-\left(\frac{x}{c}\right)^8}
\]
Simulation

- Planets at $7\lambda/d$ and $21\lambda/d$, each $10^{-10}$ brightness of host star
- 3 colours shown: 760nm (blue), 872nm (green), 1000nm (red & black)

Units: $\lambda/d$ (at 1000 nm)
Broadband

- V-band in 8 wavelengths and star stop
Error Sensitivity

- Simulation of random amplitude errors
  - random amplitude error $10^{-3}$ max.
  - pixel size $\sim 1$ cm
    (results dependent on pixel size)
Two Dimensions

- square input pupil
- 3x3 replications
- super-Gaussian apodisation
- no star stop
- 64% efficiency
- V-band
- not optimised

energy (normalised)

angle (\lambda / D)

angle (\lambda / D)

energy (normalised)
Classical Coronagraph + PR

- Square input pupil
- 8x8 replications
- Hard edged square star stop
- Idem Lyot stop
- 50% efficiency
- V-band
- Not optimised

Diagram showing energy (normalised) as a function of angle (\(\lambda / D\)).
Off axis PSF (planet)
Astrophysical Journal Letters 618:
Pupil Replication for Exo-Planet Imaging
Greenaway et al. (10 January 2005)