Generalised Phase Diversity Wavefront Sensor

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Format for this presentation

- Brief introduction to the existing Phase Diversity (PD) method.
- Limitations of the current method.
- Theory behind Generalised Phase Diversity (GPD).
- Implementation and Data Reduction
- Future research and Conclusions.

Phase Diverse Wavefront Sensing

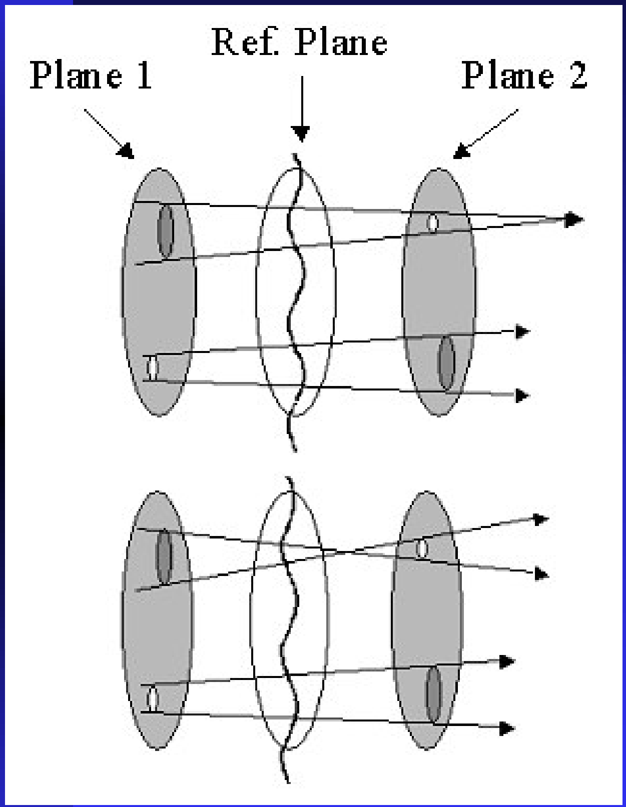


Figure 1: Two intensity planes either side of the wavefront

- DoE used to image Planes 1 & 2
- Solution of ITE gives wavefront

$$\frac{I_{\text{Plane 1}} - I_{\text{Plane 2}}}{z_1 - z_2} \sim \frac{\partial I}{\partial z}$$

$$\Psi(r) = -k \int_R dr' G(r, r') \frac{\partial I(r')}{\partial z}$$

Diffractive Optics

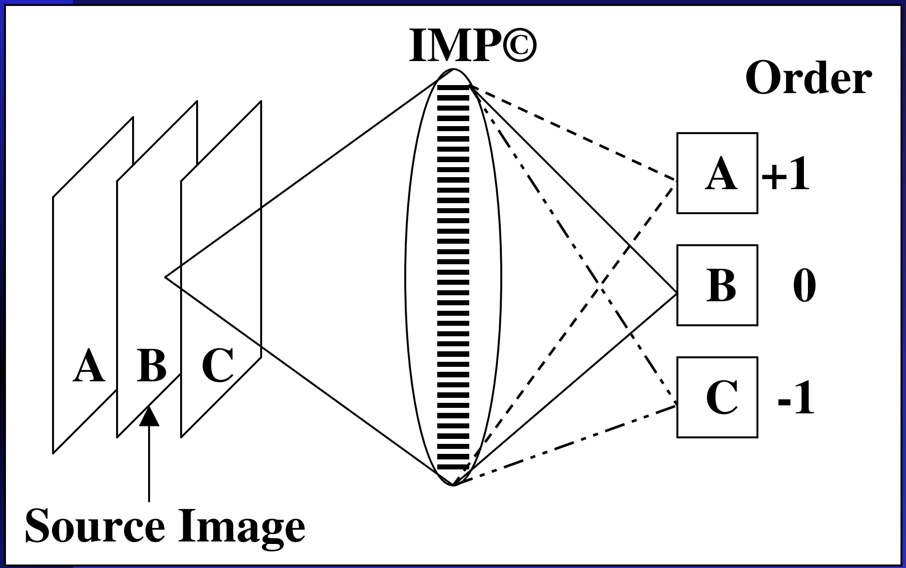
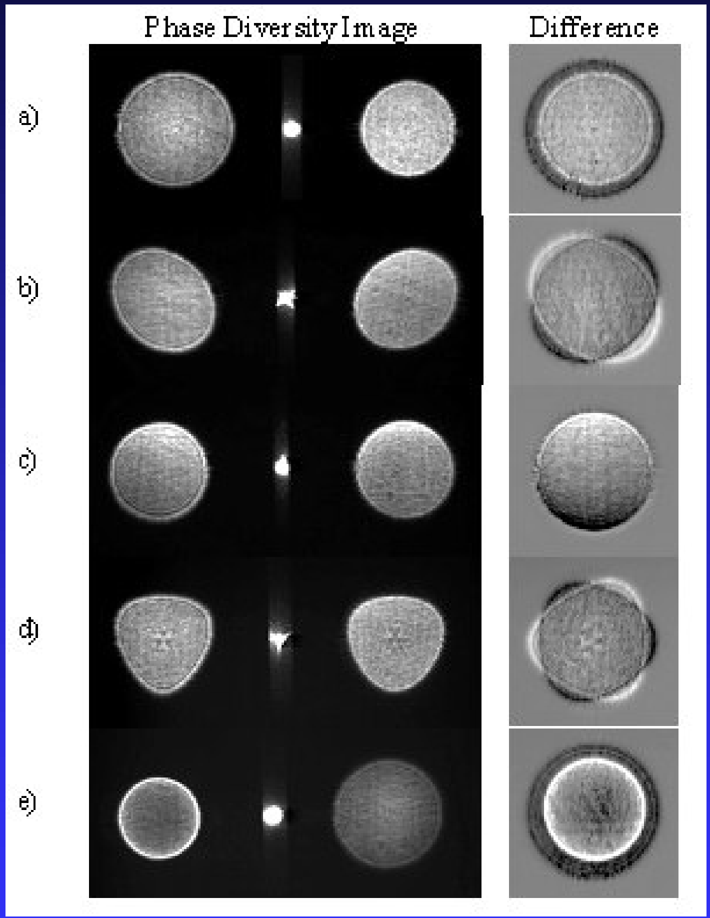


Figure 2: Shows the design of the current wavefront sensor.

Note: IMP© is a DERA (now QinetiQ) trademark

- Quadratically distorted defocus grating.
- Images of different object layers are recorded on the same focal plane.
- The plane separation and image locations are determined by the properties of the grating.

Examples of Data



•Some examples of the data seen at the focal plane.

•Easy to see the aberrations present in the data just by eye.

- •Defocus
- •Astigmatism
- •Coma
- •Trefoil
- •Spherical Aberration

Blanchard, P.M., et al., *Phase-diversity wave-front sensing with a distorted diffraction grating.* Applied Optics, 2000. **39**(35): p. 6649-6655.

Limitations

- The current Greens' function solution to the differential Intensity Transport Equation imposes several restraints on the wavefront to be reconstructed:
  - The wavefront phase must be continuous within the pupil.
  - The derivative of the wavefront phase (slope) must be continuous within the pupil.
  - The wavefront reconstruction requires computing effort and causes delay.

Generalised Phase Diversity

- GPD is required for a null sensor suitable for use with scintillated and discontinuous wavefronts.

- Images formed by convolution of the input wavefront with an aberration function (currently defocus) that has an equal but opposite aberration in the ± diffraction orders.

  - What,if anything, is unique about defocus?
  - What generic properties must an aberration function possess for use in a null sensor?
  - Can this function be optimised using *a priori* information about the wavefront to be measured?

GPD Theory - Definitions

- Complex Distribution in the entrance pupil

$$\Psi(r) = \left|\Psi(r)\right| e^{i\varphi(r)}$$

- $H(\xi)$ and $A(\xi)$ respectively represent the Fourier transforms of the real and imaginary parts of $\Psi(r)$.

$$\psi(\xi) = H(\xi) + A(\xi)$$

- $F_{\pm}(\xi)$ are the filter functions:

$$F_{\pm}(\xi) = R(\xi) \pm i.I(\xi)$$

GPD Theory - Definitions

- The detected phase-diversity intensity functions are:

$$j_{\pm}(r) = \left| \int d\xi . \psi(\xi) . \mathrm{F}_{\pm}(\xi) . e^{-i\xi r} \right|^2$$

- d(r) is the difference between the images in the ±1 diffraction order

$$\mathrm{d}(r) = \mathrm{j}_{+}(r) - \mathrm{j}_{-}(r)$$

$$\mathrm{d}(r) = 2i\left[\int d\xi \psi(\xi)\,\mathrm{I}(\xi) e^{-i\xi r} \int d\xi' \psi^{*}(\xi')\,\mathrm{R}(\xi') e^{i\xi' r} - \int d\xi \psi(\xi)\,\mathrm{R}(\xi) e^{-i\xi r} \int d\xi' \psi^{*}(\xi')\,\mathrm{I}(\xi') e^{i\xi' r}\right]$$

Symmetries of the Filter Function

- The error signal can therefore be expressed:

$$\frac{\mathrm{d}(r)}{2i} = \int d\xi\, \mathrm{H}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{H}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{A}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{A}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{A}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{A}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$

- Filter function <u>must</u> be complex valued, or d(r) will be zero $\forall\, \xi$

Same Symmetry

- If R and I are both odd, or both even:

$$\frac{\mathrm{d}(r)}{2i} = \int d\xi\, \mathrm{H}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{A}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{A}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$

- A plane wave input has constant phase. Let this phase be zero, therefore $\mathrm{A}(\xi) = 0$. The error expression above will given d(r)=0 in this case, and generate a signal for non-plane waves.

Mixed Symmetry

- In the mixed symmetry case:

$$\frac{\mathrm{d}(r)}{2i} = \int d\xi\, \mathrm{H}(\xi)\,\mathrm{I}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{H}^*(\xi')\,\mathrm{R}(\xi')e^{ir.\xi'} - \int d\xi\,\mathrm{H}(\xi)\,\mathrm{R}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{H}^*(\xi')\,\mathrm{I}(\xi')e^{ir.\xi'}$$
$$+\int d\xi\,\mathrm{A}(\xi)\,\mathrm{I}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{A}^*(\xi')\,\mathrm{R}(\xi')e^{ir.\xi'} - \int d\xi\,\mathrm{A}(\xi)\,\mathrm{R}(\xi)e^{-ir.\xi}\int d\xi'\,\mathrm{A}^*(\xi')\,\mathrm{I}(\xi')e^{ir.\xi'}$$

- When $A(\xi)=0$ this expression will still generate a signal. Therefore it is unsuitable for use in a wavefront sensor.

Necessary & Sufficient Conditions

- *Sufficient Condition*: The difference (d(*r*)) between two aberrated images is null if the input wavefront has Hermitian symmetry (I.e. is purely real) and is non-null for non-plane wavefronts.

- *Necessary Conditions:*
  - The filter function must be complex.
  - Mixed symmetries (of R and I) must not be used

Error Direction & Location

- If the sense of the error reverses (changes sign) the sign of A(ξ) will also be reversed $(A(\xi) = -A^{*}(-\xi))$
- The relationship between the error signal and the wavefront error is non-linear.
- The location of the wavefront error can be identified with the position that a(r) is non zero.
- Heuristic experience with the existing PD wavefront sensor suggests that both the sense of direction and location of the error should be preserved well enough for use as a null sensor.

Implementation

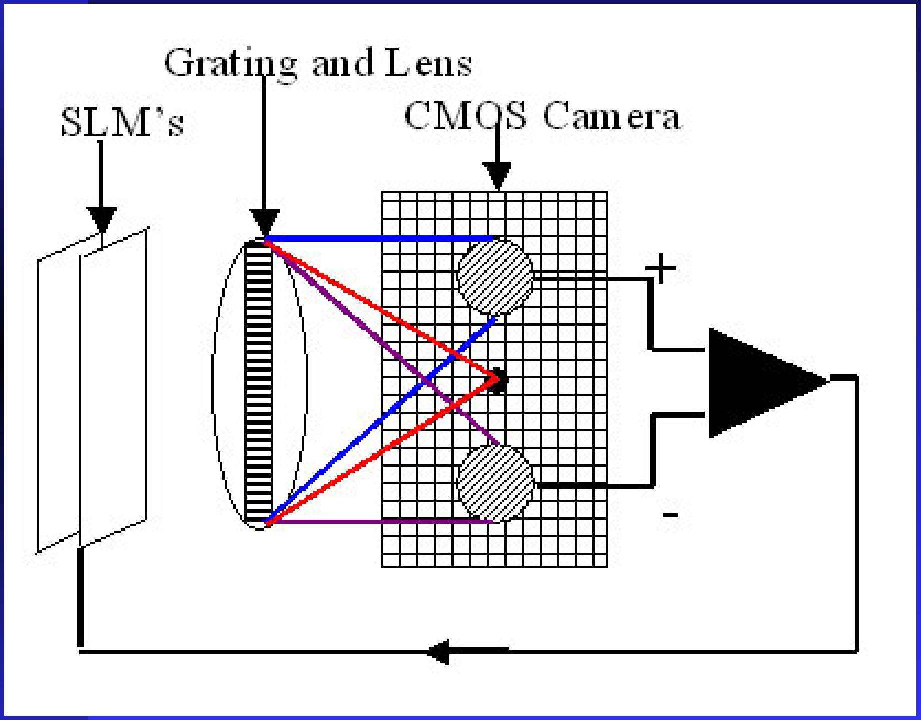


Figure 3: A suggested Compact AO System (CAOS)

- Common path aids compact design
- SLMs provide modulation.
- DoE combines phase diverse data and corrected image.
- CMOS camera

Data Reduction

- Phase reconstruction is possible using an algorithm based on error-reduction
- Simulations have been conducted to validate the theory presented here. Some of these results will be presented by my colleague Dr Zhang this afternoon.
- Full reconstruction is un-necessary when operating as a null sensor.

Future Research

- Experimental validation of the theory:
  - Manufacture of gratings for use as GPD filter functions and also to create test wavefronts with known aberrations.
  - Construction of a wavefront sensor based on these principles, using these gratings.
  - Study of optimisation when *a priori* information about the wavefront aberrations is available.
  - Implementation on a real system (WFCAM)?

Conclusions

- There is a need for a more generalised approach to PD wavefront sensing, to overcome the limitations of the current method.
- We have discovered necessary and sufficient conditions that a filter function must possess for use in a GPD based null sensor.
- Simulations that confirm this theory have been conducted.
- We have demonstrated that a compact AO system could be built based on these principles.
- Experimental testing and optimisation is to be conducted.