Wavefront Sensing @ Heriot-Watt

What we are doing and why!

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Talk Outline

- Brief description of the GPD wavefront sensor
- The Small Angle Expansion
  - Why? What? How?
  - Issues
  - Preliminary Results (Beijing)
  - Current work
- Where do we go from here?

PD with Diffractive Optics

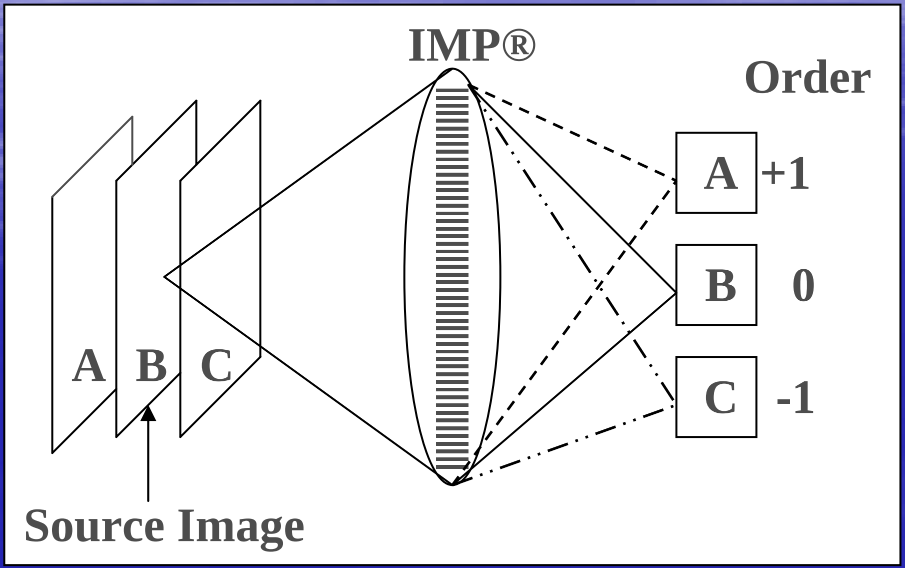


Figure 1: Shows the design of the current wavefront sensor.

Note: IMP® is a DERA (now QinetiQ) trademark

- Quadratically distorted defocus grating.
- Images of different object layers are recorded on the same focal plane [1].
- The plane separation and image locations are determined by the properties of the grating.

GPD ~ How we get error signals

- Diffraction Grating – applies diversity to the input wavefront.
- Intensity images in the ±1 orders contain the wavefront convolved with the diversity function.
- We use the difference between these intensity images to form the error signal d(r).
- When the input wavefront is unaberrated the ±1 images are identical and d(r) is zero.

Small Angle Expansion ~ Why?

- We need an algorithm to retrieve the phase of the input wavefront – preferably this will be an analytic solution.
- We want an algorithm that works for any allowable diversity function (unlike the Green's function or Nugent algorithm.)
- We want the algorithm to be robust, capable of dealing with scintillated and discontinuous wavefronts.

The Error Signal

- Formed by the difference between the intensity images in the ±1 diffraction orders.
- Full details of the error equation and how we derived it can be found in Optics Letters **29**(23): p. 2707-2709 (2004) [2]
- The small angle approximation has been used to linearise this equation and allow us to solve for the phase of the unknown wavefront.

Small Angle Expansion ~ What?

- We will use this familiar mathematical trick to derive a solution for the retrieved phase.

- The derivation itself is not difficult, here I will restrict this talk to a brief summary of the most important parts.

And now for a little bit of maths….

A little bit of maths…

- Let a wavefront be given by:

$$\psi(r) = A_0 \exp\left(i\varphi(r)\right)$$

- The Small Angle Approximation [SAA]:

$$\text{When } \varphi(r) \approx 1$$

$$\exp\left(i\varphi(r)\right) \approx 1 - \varphi^2(s) + i\varphi(s)$$

A little bit of maths…

- Consider the wavefront as being made up of a real and an imaginary part:

$$\psi(r) = h(r) + ia(r)$$

- Then under the SAA:

$$h(r) = \sqrt{I_0}\left[1 - \varphi^2(r) + O\left(\varphi^4(r)\right)\right]$$

$$a(r) = \sqrt{I_0}\left[\varphi(r) - O\left(\varphi^3(r)\right)\right]$$

A little bit of maths…

Ignoring higher order terms, and assuming no/moderate scintillation:

$$h(r) \approx \sqrt{I_0}$$

$$a(r) \approx \sqrt{I_0}\left[\varphi(r)\right]$$

A little bit of maths…

- The error signal is the result of the convolution of the input wavefront with the blur function {the FT of the diversity filter function}.

- We can show (and will soon publish):

FT Phase Transfer Function

Error Signal

$$\frac{d(r)}{4h(r)} = \int d\xi \Phi(\xi) T(\xi) e^{-ir\xi}$$

Real part of the input wavefront

FT Wavefront phase

Solving for the phase:

$$\Phi(\xi) = \frac{D_h(\xi)}{T(\xi)}$$

Where $D_h(\xi) = \Im\left[\frac{d(r)}{4h(r)}\right]$

Data Conditioning

What does this mean?

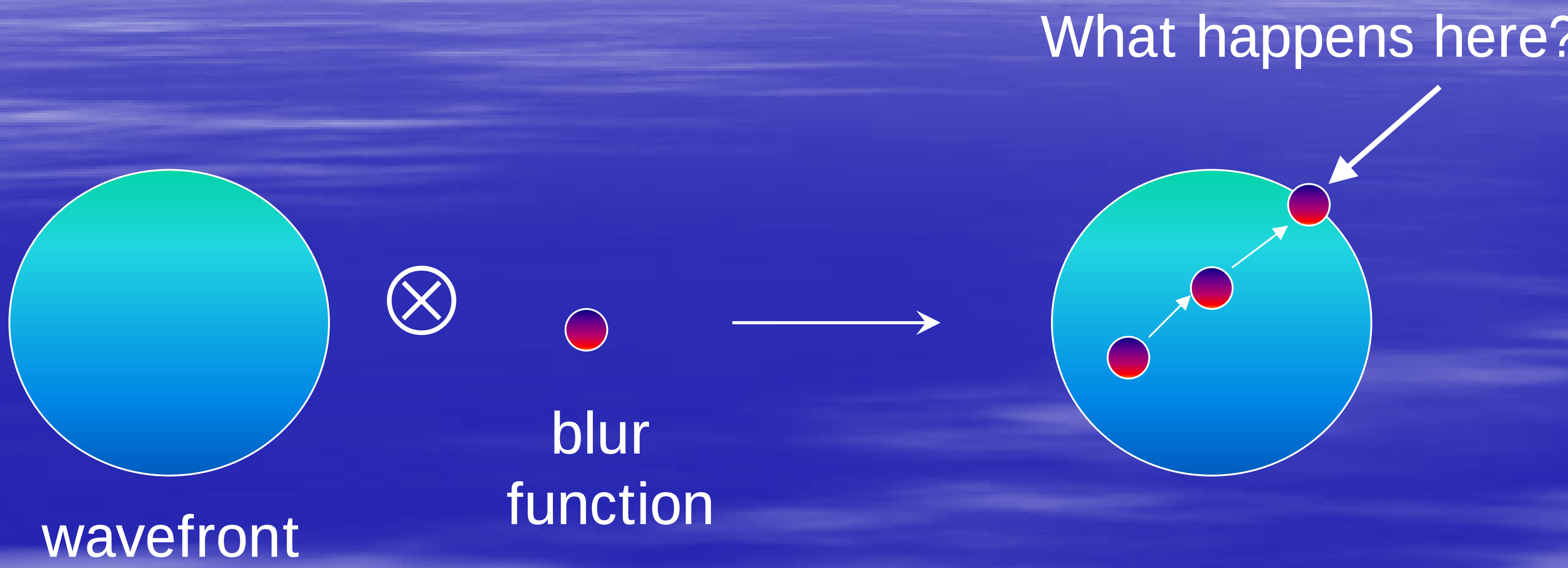
- We can reconstruct the phase using an error signal that we measure, and a Phase Transfer Function (PTF)
- The PTF is defined by the diversity function that we choose
- We can choose/design a PTF to suit specific applications!

Properties of the SAE

- We know that the error signal is the result of the convolution of the wavefront phase with the PTF/blur function

What does this look like and what does this tell us about the SAE?

The Convolution Process



Issues

1. What exactly do we mean by “small” angle?

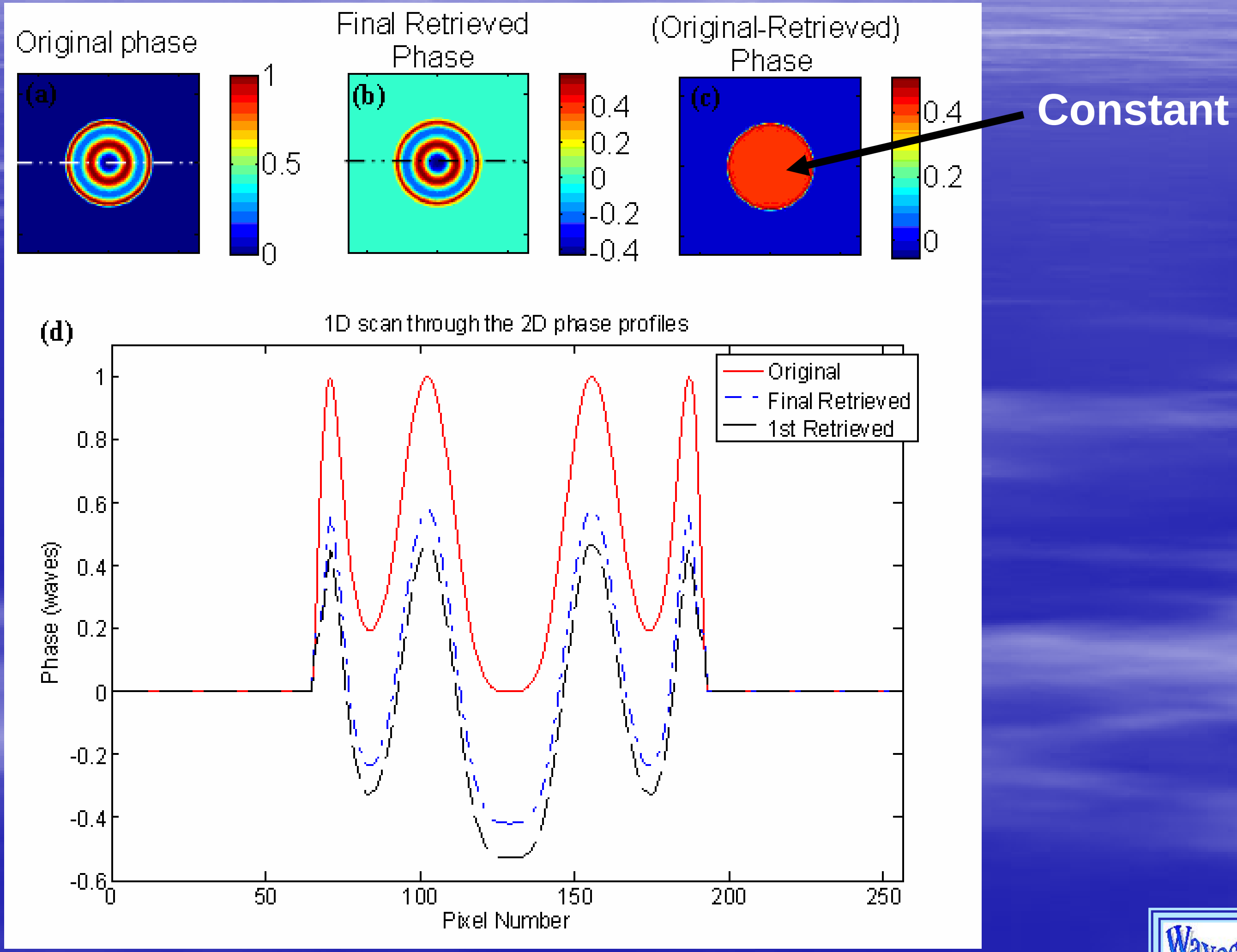
2. What happens at the boundary?

3. What happens if the data is noisy?

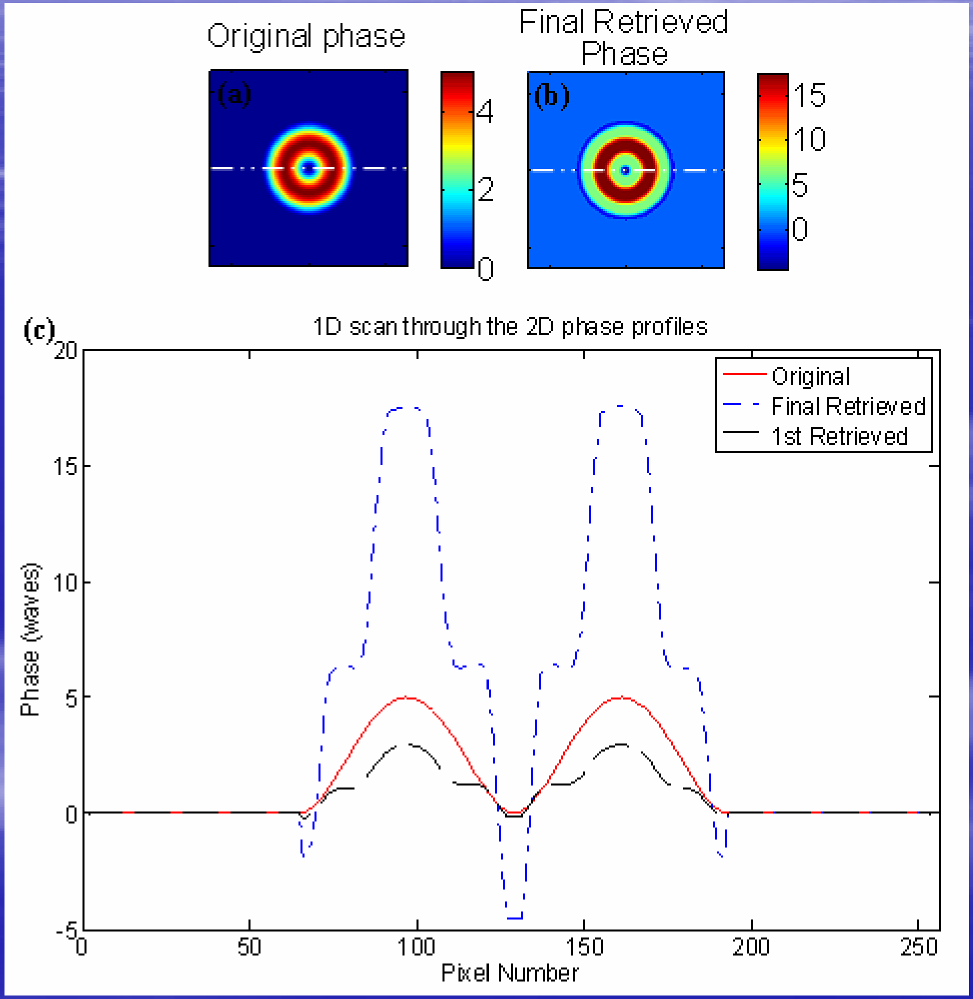
“Small” Angles

- The PV of the wavefront error is not the problem!
- The PV step and slope over the area of the blur function is the issue, and should obey SAE.
- However, we will still encounter problems at the edges!

High slope example - 1



High slope example - 2



Issues

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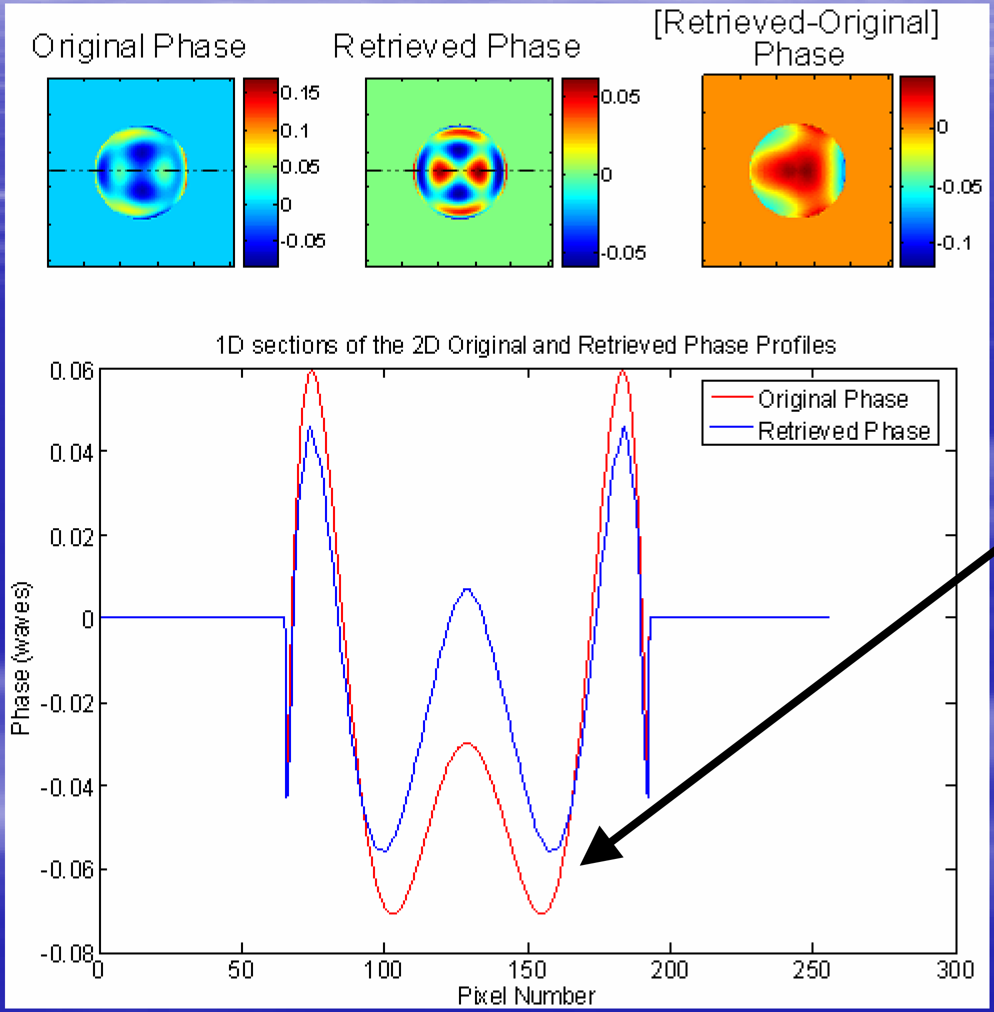
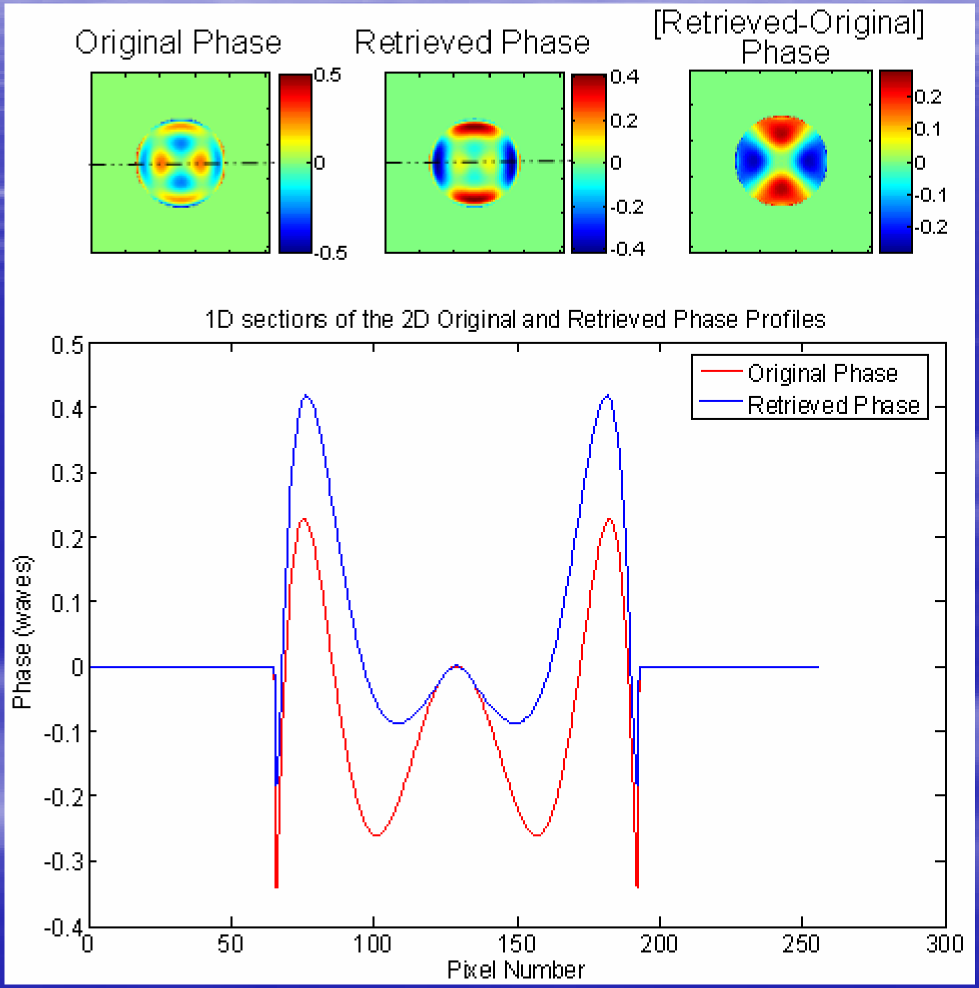
Hard Edge Example -1

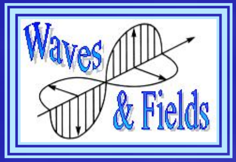


The shape isn't bad but there are large amplitude errors

Hard Edge Example -2



SAME PROBLEM!



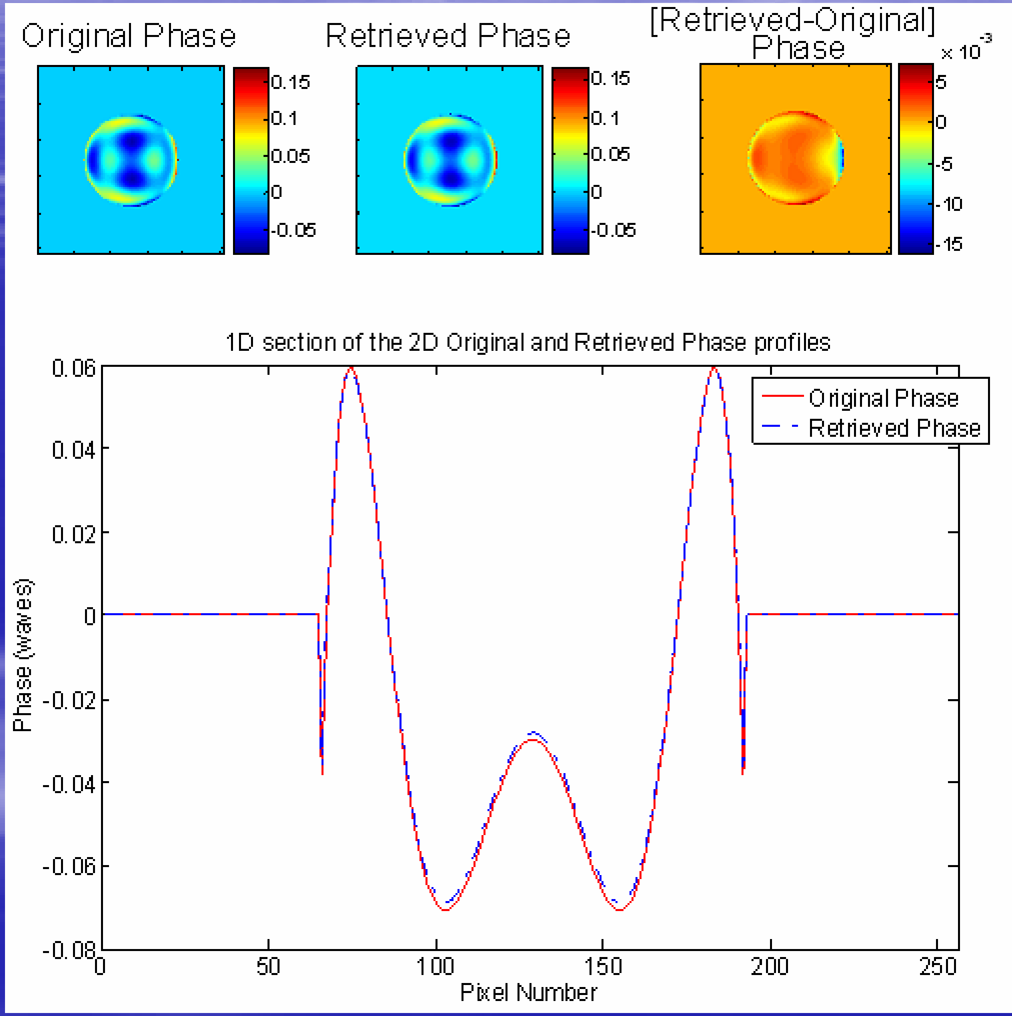
Hard Edge Simulations

- In all the simulations conducted using a hard edge pupil the general retrieved shape was good, but there were large amplitude errors.
- This could be indicative of a boundary value problem….

Using a Soft Edged Pupil

- In the hard edge simulations we observed amplitude errors which may indicate a boundary problem
- When the blur function reaches the edge, what happens if it sees a gradual slope, instead of a sudden drop?

Soft Edge Pupil Simulations



Here we see the same example, but this time the fit is much better!

Issues

1. What exactly do we mean by "small" angle?

2. What happens at the boundary?

3. What happens if the data is noisy?

Regularisation Issues

- Earlier we saw that the wavefront phase can be retrieved using the following equation:

$$\Phi(\xi) = \frac{D_h(\xi)}{T(\xi)}$$

$$\text{Where } D_h(\xi) = \Im\left[\frac{d(r)}{4h(r)}\right]$$

What happens when the data is noisy?

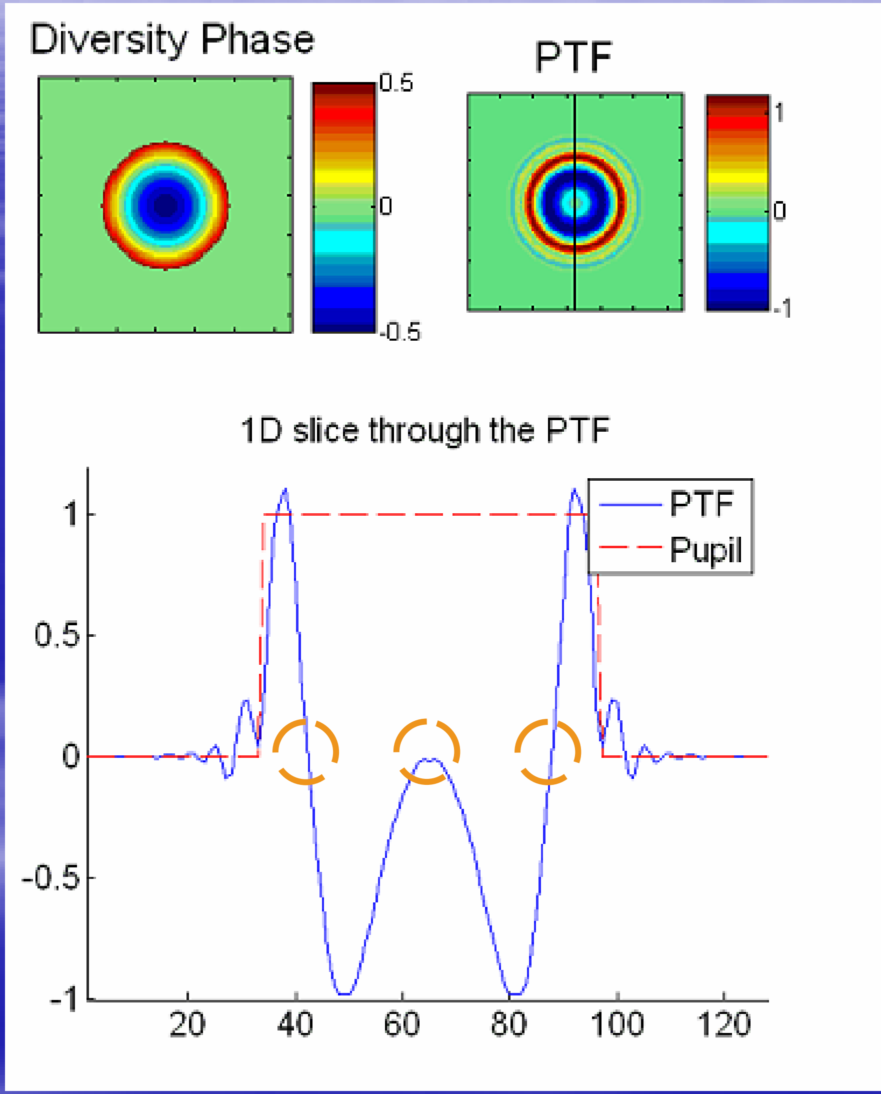
Noisy Data….

$$\Phi(\xi) = \frac{D_h(\xi) + N_h(\xi)}{T(\xi)}$$

Classic regularisation problem!

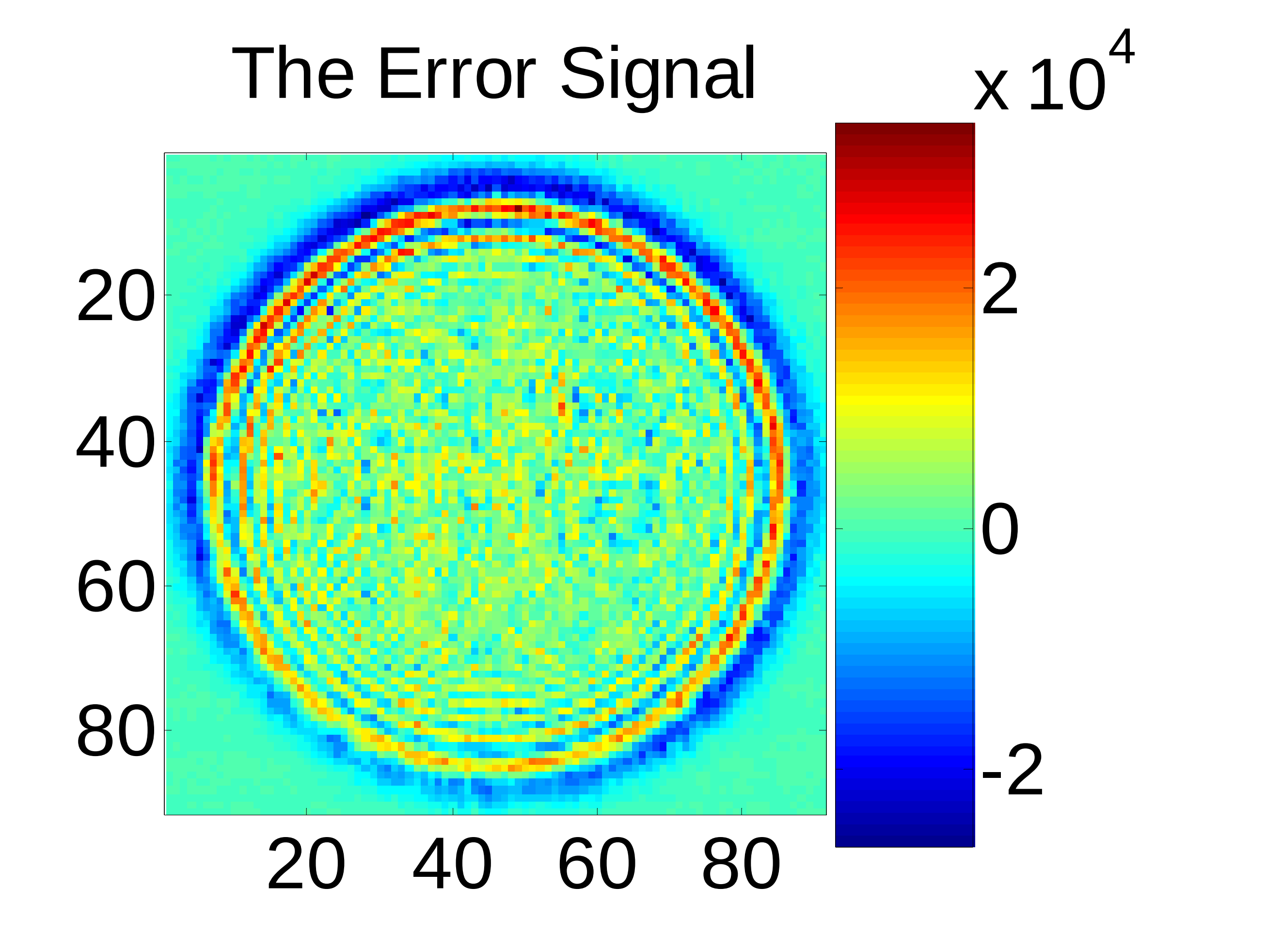
As $T(\xi) \rightarrow 0$, $D_h(\xi) \rightarrow 0$ but $N_h(\xi) \rightarrow \infty$!!!

Properties of the PTF

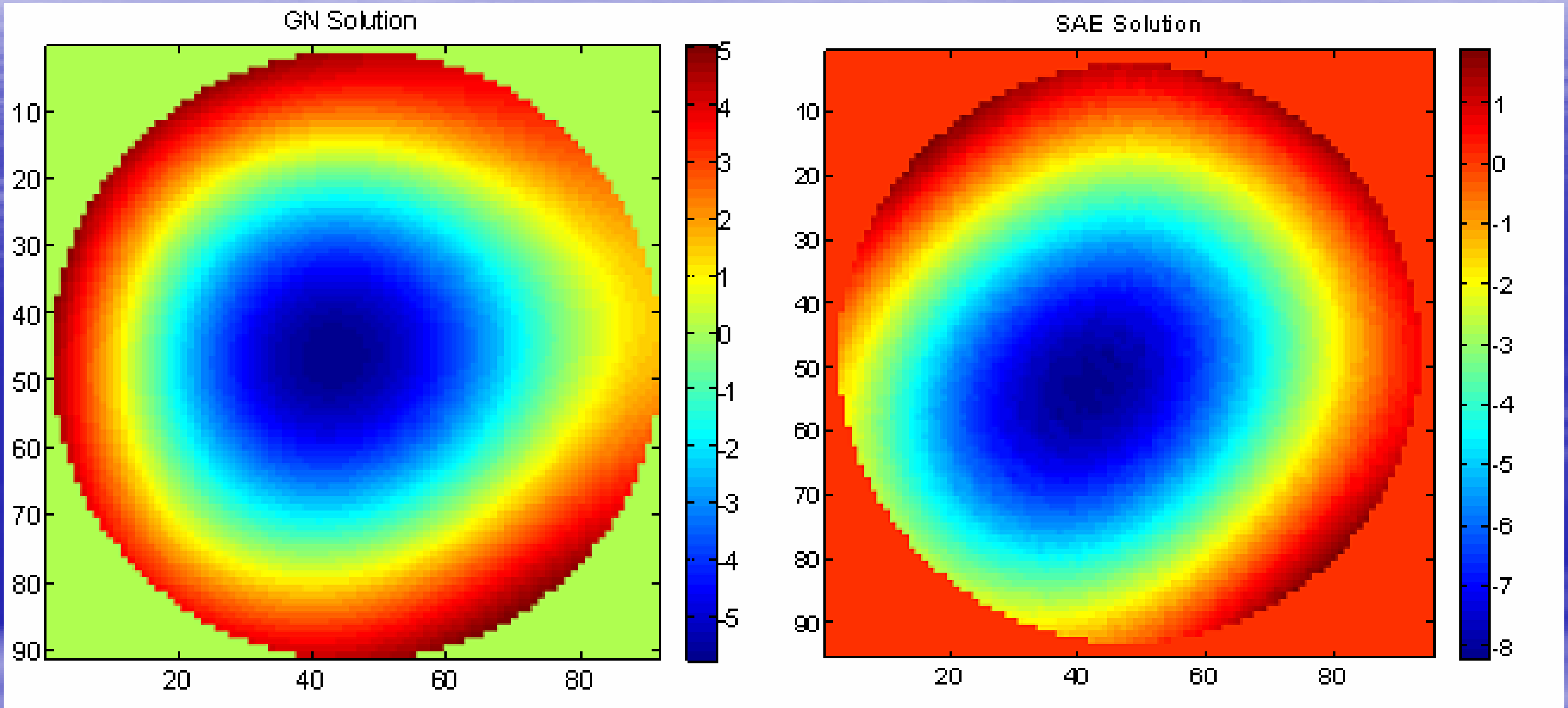


Regions where the Phase Transfer Function goes to zero cause regularisation problems!

Preliminary Results (Beijing)

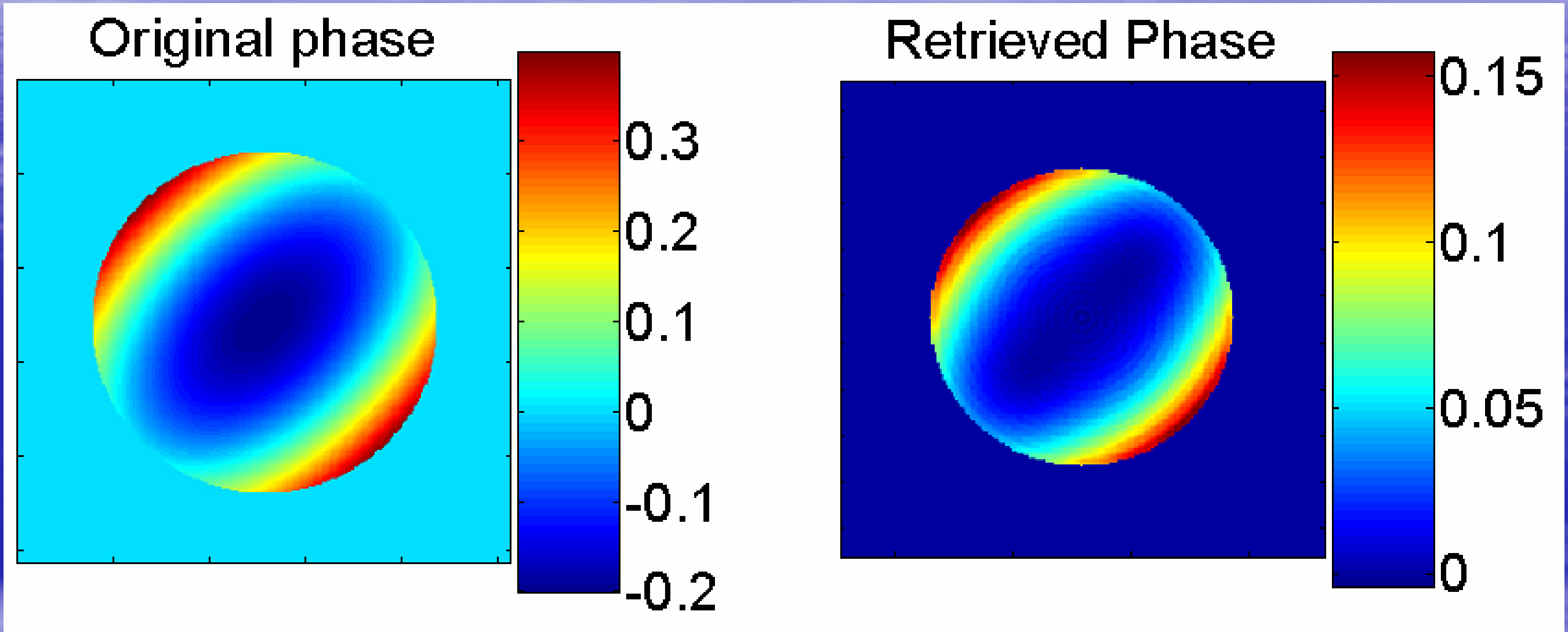


Preliminary Results



- The direction of asymmetry in the SAE solution accords better with the asymmetry of the error signal.

Simulated (noiseless) Results



• The shape and orientation of the phase profile are correct, but there is as yet an unexplained scaling error contained in the SAE

Other Issues

Boundary Problems:

- Can we get a better result by reconstructing the imaginary part of the wavefront instead?

Data Conditioning

- What is the best function to use?

$$D_h(\xi) = \Im\left[\frac{d(r)}{4h(r)}\right], \quad h(r) \approx \sqrt{I_0}$$

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