Pupil Replication

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PREDRIS

• Pupil Replication for Extreme Dynamic Range Imaging Spectroscopy
• Start: 1 April 2005
• Two years, funded by PPARC
• Publication: Pupil Replication for Exo-Planet Imaging; ApJL, 10 January 2005 (referenced in Nature).
• This presentation: includes new unpublished results.
Overview

• Introduction to Pupil Replication
• Characterization and analysis
• Error Assessment
• Experimental proof of principle
• Simulation of a Coronagraph
• Comparison with Hyper Telescope principle
Context

• Exoplanet indirect detection since 1995
• ESA Cosmic Vision 2020 (April 2005):
  – fundamental physics
  – cosmology
  – solar system
  – planetary formation, life, exoplanets
  – => direct detection of exoplanets
  – => proposals due early 2007
Numbers

- Dynamic range starflux / planetflux:
  - $10^{10}$ (visual, reflected), $10^6$ (infrared, thermal)

- A planet at 1 AU around a star at 20 pc seen with 3 m telescope
  - at 36 λ/d at 600 nm (stardisk => 0.2 λ/d)
  - at 2 λ/d at 10 micron
  - flux: one hour, V-band, efficiency 50% =>
    - ~500 photons (planet around 0 magnitude star)
    - ~.05 photons (planet around 10 magnitude star)
Star flux problem

- Flux from host star dominates exoplanet:
- Techniques to suppress scattered host flux:
  - coronography
  - pupil apodisation
  - interferometry

Diffraction limited PSF,
x-axis in $\lambda/d$
Inner Working Angle

- Inner working angle (IWA): the minimum angular distance from the star at which a planet can be detected.
- All techniques need image-plane masks
  - mask size is typically several $\lambda/D$
  - side-lobe suppression often increases star image
  - increasing IWA makes earth-like detection harder
- $\Rightarrow$ decrease size of star image
  $\Rightarrow$ pupil replication
Pupil Replication with Apodisation

energy (normalised)

angle (\lambda / D)

energy (normalised)

angle (\lambda / D)
Pupil Replication

- Axial wavefront is continuous (blue), like larger telescope.
- Non-axial wavefront is discontinuous (red), and appears like a blazed grating.
- Etendue is preserved (pupil area * solid view angle)
Expected Results

- Consider 3-fold replication in 1-d
- Axial pupil wavefront is 3x wider and 3x fainter
- Image of unresolved axial star is 3x narrower and 3x brighter
- Sidelobes are 3x fainter
- Image stop to remove star flux can be smaller
Expected Limits

- Violation of homothetic mapping
  - position and orientation of the replicated images is not an exact scaled copy of the unreplicated image

- => Non-isoplanatic imaging
  - PSF dependent on the viewing direction.

- => Requirement:
  - Maximum angular diameter of star + telescope pointing error < some limit: $\theta$
Example

- General criterion: max wavefront error < $\lambda/4$.
  - => For N-fold replication of aperture diameter D and at wavelength $\lambda$:

- Example:
  - 2.5m diameter pupil
  - 800nm wavelength
  - 3-fold pupil replication
  - $\theta < 5$ mas

- At angles smaller than $\theta$ the image quality should be ‘good’ (includes pointing error).

$$\theta < \frac{\lambda}{4D(N-1)}$$
Simulation

- Broken line is un-replicated image (diffraction limited)
- Solid line is replicated image
- Replicated image is modulated by un-replicated image, as expected

y: log(int)  x: mas

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Broadband

- Single pupil, 1000 nm (black)
- 3x Replicated pupil
  - 1000 nm (red)
  - 872 nm (green)
  - 760 nm (blue)
Analysis (1)

in 1 dimension, plane wave on axis, hard edged pupil:

\[
\text{wave + pupil:} \quad \text{imaged:}
\]

\[
\sin(x)/x = \text{sinc}(x) \quad \text{sinc}(x) (1+2\cos(x))
\]
Analysis (2)

- Input wave at angle $\alpha$, $d =$ pupil diameter
- $A =$ amplitude constant

\[
wave(x) = Ae^{\frac{2\pi(x-\sin(\alpha))/\lambda}{\lambda}}
\]

- Amplitude of imaged plane wave $= T$

\[
T = A \text{sinc} \left( 2d \frac{\pi(x - \sin(\alpha))/\lambda}{\lambda} \right)
\]

- PSF $= |T|^2$
Analysis (3)

• Three replications:
  – single pupil convolved with 3 delta functions
  – Fourier(cosine) = 2 delta functions
  – => sinc multiplied in Fourier domain (image) with $1 + 2\cos(x)$

$T = A(1 + 2\cos(2d\pi x/\lambda))\text{sinc}(d\pi(x - \sin(\alpha))/\lambda)$

– Note: Cos( ) is not dependent on angle
Two dimensions

- Square pupil (seamless replication) then: 2-D amplitude $T$

$$T = T_x T_y$$

- $T$ for two replications 1 and 2
  - 1 is on axis, 2 is adjacent

$$T = T_{x1} T_{y1} + T_{x2} T_{y2}$$

- errors imposed on 2
Errors

• Pupil 1: \( T_{1x} \) and \( T_{1y} \) are sinc functions
• Pupil 2 has errors in x (and y) direction:
  – shift \( s \) (and shear \( h \)):
  – piston \( p \)
  – tilt \( f \) (and tip \( g \))
  – \( T_{2x} \) (and similar for \( T_{2y} \)):

\[
Ae^{i2\pi \left(p + \frac{x}{\lambda}(d+s)\right)} \text{sinc} \left(d\pi \left(\frac{x - \sin (\alpha)}{\lambda} - f\right)\right)
\]
Evaluation

• 2-D analyses; horizontal crosssection
• If PSF(no error) - PSF(with error) < $10^{-10}$
  – example: shift = d $10^{-5}$ => 1 micron for 1 cm pupil
  – difference image =>

![Graph showing 10log of normalised intensity vs. off-axis angle (λ/d)](image)
Examples

• Similar results below $10^{-10}$ for
  – shear = $10^{-5} \, d$; piston = $10^{-6} \, \lambda$; tip = $10^{-5} \, \lambda/d$; tilt = $10^{-5} \, \lambda/d$; jitter = $10^{-6} \, \lambda/d$

• No apodisation or star stop applied.

• Is this peculiar to PR ? =>
  – Error sensitivity assessed this way is for single pupil of the same order of magnitude.
Software Simulations

- 2-D; horizontal crosssections shown
- Simulations of errors due to
  - shift, shear, piston, tip, tilt, jitter
  - agree with analysis within $10^{-14}$
- Further simulations:
  - Rotation error, (de-)magnification error,
    random amplitude errors, random phase errors.
  - Extended sources
Simulation: Procedure

- Setup: two square pupils like in analysis
  - wave - pupil - errors - FT - grating - FT\(^{-1}\) - pupil
    - apodisation - FT - star stop - sampling correction - modulus\(^2\) - store
  => this for each point source and wavelength and sum the results
  - Adapt for each wavelength: size of pupils, of apodisation, of star stop, of grating.
  - NB: maximum # pixels limited; sampling correction important; accuracy \(~10^{-14}\)
Example

- Rotation error, max = 21 degrees (magenta), and no error (black), diagonal crosssection.

- Similar results for (de-)magnification error.
- Random errors addressed later.
Extended source

- Simulation using 500 point sources in broadband (8 wavelengths), source = 5λ/d
- The cosine term marks the profile (?)
  - black: single pupil
  - red: 3 replications
  - equal normalisation
- Important when star > λ/2d
  (sun at 20 pc with 9 m telescope at 600 nm)
Half-time

• So far:
  – Pupil Replication as such
  – Analysis, error behaviour

• To come:
  – Experiment
  – Coronagraph
  – Hyper telescope?
Experiment

- A first experiment to verify principle
- Interferometer beam:
  - 2 coherent beams
  - Monochromatic
  - Circular pupils with Gaussian beams
- Effect in one dimension only

Adjust shear
PSF, pupils: 50% overlap

simulation

experiment

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H:\Work\Education\Erwan\50% OVERLAPPING\PSF.bmp

H:\Work\Education\Erwan\50% OVERLAPPING\CPSF.bmp
Cross-sections

Blue: simulation, unreplicated
Red: simulation, replicated
Black: experimental
PSFs, pupils: no overlap

simulation

experiment

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Cross-sections

Blue: simulation, unreplicated

Red: simulation, replicated

Black: experimental
Replication

- Use 1 beamsplitter and mirrors

- Equal:
  - # reflections in each arm
  - polarisation
  - optical path length

- Adjust last prism to vary replica separation
  => no shift error

- Can be cascaded
- Can be monolithic
Pupil Apodisation

- Used simple not optimised function for attenuation $B$:
  - Super Gaussian
  - $x = \text{off-axis angle}$
  - $c$ is adjusted to attenuate by $10^{-4}$ at pupil edge (both un- and replicated)

- Expect to
  - broaden the image core
  - reduce side-lobes

$$B(x) = e^{-\left(\frac{x}{c}\right)^8}$$
Simulation

- **Unapodised**
  - Black: unreplicated
  - Green: 3-fold replication
- **Apodised**
  - Red: unreplicated
  - Blue: 3-fold replication
- **Efficiency:**
  - 65% throughput
- **Sidelobes suppressed**

Units: $\lambda/D$

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Off-axis effects

- Off-axis (planet) images will be distorted:
  - Wavelength dependent => chromatic effects.
  - Will these obstruct spectroscopy?
  - What other effects will this have - SNR?

50 mas

100 mas

150 mas

y: log(int)   x: mas

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Simulation

- Planets at $7\lambda/d$ and $21\lambda/d$, each $10^{-10}$ brightness of host star
- 3 colours shown: 760nm (blue), 872nm (green), 1000nm (red & black)

Units: $\lambda/d$ (at 1000 nm)

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Star Stop added
– $5\lambda/d$ half-width solid star stop in image plane
– second pupil added, no Lyot stop
– Unreplicated: solid red, replicated: solid blue
– right: three wavelengths (coloured)
Coronagraph

- Broadband (V-band in 8 wavelengths)
Sensitivity: amplitude

- At this accuracy, amplitude errors matter
- Simulations suggest that better than 0.1% is required to image exo-earths
- Simulation:
  - random amplitude error $10^{-3}$ max.
  - pixel size $\sim 1$ cm
Pixel size

- 10-3 random amplitude errors, with 25, 251, 2501 pupil pixels (100, 10, 1 mm/pixel)
Sensitivity: phase

- Higher sensitivity to phase errors
- Simulation suggests that better than $\lambda/10000$ is needed to image exo-earths
- Pixel size 1 cm

Random phase error max.

- $10^{-3}\lambda$
- $10^{-4}\lambda$
- $10^{-5}\lambda$
Hyper Telescope?

- Pupil Replication \( \neq \approx \neq \) Hyper Telescope?
  - Assessment of the hyper telescope principle
  - using analysis like before
  - simplified situation
  - characterisation only
  - adding to discussion...
Overview

On axis: Pupil Replication = Hyper Telescope

Off axis:

pupil replication  hyper telescope
Hyper Telescope

• 1-D, 3 pupils with equal spacing h between the telescopes and joined:

\[
T = A \left(1 + 2 \cos \left(2 \pi \left(x + \left(\frac{h}{d} - 1\right) \sin (\alpha)\right)/\lambda\right) \right) \text{sinc} \left(\frac{d \pi (x - \sin (\alpha))/\lambda}{\lambda}\right)
\]

– => similar to pupil replication but:
– => cos( ) now dependent on angle
– on axis (star light suppression) equal to PR
HT Analysis

- PSF: $\alpha = 0.3 \lambda/d$ of single pupil,
  $h = 2d$ (3d centre to centre)

Pupil Replication (PR)  Hyper Telescope (HT)
HT Analysis

- Same as last slide but:
- PSF: $\alpha = 1.6 \lambda/d$ of single pupil,

Pupil Replication (PR)  Hyper Telescope (HT)
Analysis

- **HT-PR:**
  the hyper telescope is replicated 3 times:

\[
T_{HT-PR} = \frac{T_{HT}}{3} \left(1 + 2 \cos \left(6\pi d \left(x - \sin(\alpha)\right)/\lambda\right)\right)
\]

- **PR-HT:**
  the 3 replications are made in each of the hyper telescopes:

\[
T_{PR-HT} = \frac{T_{HT}}{3} \left(1 + 2 \cos \left(6\pi d \left(x + (h/d - 1) \sin(\alpha)\right)/\lambda\right)\right)
\]
HT-PR or PR-HT

- 3x3 pupils
- $\alpha = 0.3 \ \lambda/d$
$H = 30 \, d, \, \alpha = 0.01 \, \lambda/d$
Evaluation?

• Based on this analysis only:
• Options:
  – pupil replication
  – hyperteleoscope
  – combined
  – other...
• Criteria:
  – on-axis behaviour: star suppression
  – off-axis behaviour: planet detection
Information


– Analysis of Pupil Replication (to be published).

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