Generalised Phase Diversity – Initial Tests

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Outline

- Numerical experiments to validate theory for generalised phase diversity.

- Description of error reduction algorithm.

- Simulation showing possible optimisation using *a priori* information

- Conclusions
The necessary and sufficient conditions for a null wave-front sensor, which was presented in the morning by Heather Campbell, are as following

**Sufficient Conditions:**

The difference between two aberrated diversity images is null if the input wave-front is plane wave and is non-null for non-plane wave-fronts

**Necessary Conditions:**

The filter function must be complex. Mixed symmetries of the filter function must not be used
A suggested Compact AO System (CAOS)

- SLMs provide modulation.
- DoE provides phase-diverse data and image wave-front for correction by SLMs
- CMOS camera
Numerical Experiments for validating Theory

1 Plane wave input
\[ \exp(j \times 1.4 \pi) \]

a) Mixed symmetry phase diversity:
\[ z_2^0 + z_3^{-3} \]
(Defocus+Astigmatism)

Difference value for plane wave input is not zero. So mixed symmetry phase diversity is not suitable to a null sensor (Necessary Condition)
Numerical Experiments for validating Theory

b) Even symmetry phase diversity:
\[ z^0 \]

Difference value for plane wave input is zero. So even symmetry phase diversity satisfies the sufficient condition for a null sensor

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Numerical Experiments for validating Theory

2 Non-plane wave input : \( \exp(j \cdot 1.4 \cdot \pi \cdot Z) \)

Even symmetry phase diversity:
\( z_0^{10} \) (Zernike)

The difference between two aberrated diversity images is non-null for non-plane wavefronts (sufficient condition)

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Input wave-front with $\pi$ discontinuity

(black -1.3*$\pi$; white -0.3*$\pi$)

This real wave-front is unimportant for astronomy but potentially important when using the wave-front sensor in metrology applications.

Even symmetry phase diversity: $z_2^0$

This is one of results derived from theory.
Error Reduction Algorithm for Generalised Phase Diversity

\[ A \exp(j\Phi) \]

\[ \Phi_1 = \Phi + \delta\Phi \]
\[ P_1 = A \exp(j\Phi_1) \]
\[ \tilde{P}_1 = \sqrt{PSF_1} \exp(j\Theta_1) \]
\[ P'_1 = A \exp(j\Theta_1) \]
\[ P'_1 = A \exp(j\Theta_1) \exp(-j\delta\Phi) \]
\[ \Phi = \text{angle}(P'_1) \]

Add Diversity
Propagation
Apply Observed PSFs
Inverse Propagation
Apply Pupil Mask

\[ \Phi_2 = \Phi - \delta\Phi \]
\[ P_2 = A \exp(j\Phi_2) \]
\[ \tilde{P}_2 = \sqrt{PSF_2} \exp(j\Theta_2) \]
\[ P'_2 = A \exp(j\Theta_2) \]
\[ P'_2 = A \exp(j\Theta_2) \exp(j\delta\Phi) \]
\[ \Phi = \text{angle}(P'_2) \]

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Definition of the error metric function

The error metric function used in simulation:

\[ RMSE = \frac{[\sum (I_+ - \tilde{I}_+)^2 / \sum (I_+^2) + \sum (I_- - \tilde{I}_-)^2 / \sum (I_-^2)]}{2} \]

where \( I_+ \) and \( I_- \) represent measured intensity data in the +1 and -1 diffraction order.

\( \tilde{I}_+ \) and \( \tilde{I}_- \) represent an estimate of the phase diversity data intensity from the reconstructed wave-front.

In simulation, we are comparing error reduction algorithm and looking for the fastest, most robust convergence properties.

Development of analytic methods for data reduction is ‘in hand’
Simulation Results for possible optimisation using a priori information

Input wave-front: defocus+astigmatism +coma+trefoil+spherical aberration.

Diversity filter: $z_2^0$

(Defocus)

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Simulation Results (continued)

Input wave-front: defocus + astigmatism + coma + trefoil + spherical aberration

Diversity filter: $z^0_4$
(spherical aberration)

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Simulation Results (continued)

Input wave-front: defocus + astigmatism + coma + trefoil + spherical aberration.

Diversity filter: $z^{0}_{10}$

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Comparison for three diversity cases

Shows that $z^0_{10}$ is marginally better than defocus, although initial convergence is slower.

However, this does illustrate that different diversity kernels can be used to achieve different results.
Conclusions:

We used numerical experiments to validate theory for the necessary and sufficient conditions for construction of a null wave-front sensor based on the principles of a generalisation of phase-diversity wave-front sensing.

So far we find no deviations numerical experiment results from theoretical results.

Describing generalised error reduction algorithm.

Simulation results illustrated that the generalised wave-front sensor can be optimised to exploit a priori information in order to maximise sensitivity of wave-front sensor.