Error reduction algorithm applied in fractional Fourier domain

Adaptive Optics Group

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1. Error reduction algorithm(In FRFT domain)

- Let Ain and Aout be the amplitudes in the spatial and FRFT domains, respectively, and φ_{in} and φ_{out} the phases in the spatial and FRFT domains, respectively

$$Ain*\exp(\varphi in) \underset{IFRFT}{\overset{FRFT}{\rightleftarrows}} Aout*\exp(\varphi out)$$

- For error reduction algorithms, two functions (1)Ain and Aout(corresponding to single-intensity measurements) or (2) φ_{in} =0(real object) and Aout (corresponding to double-intensity measurements) are known, we need to find the other two corresponding functions.

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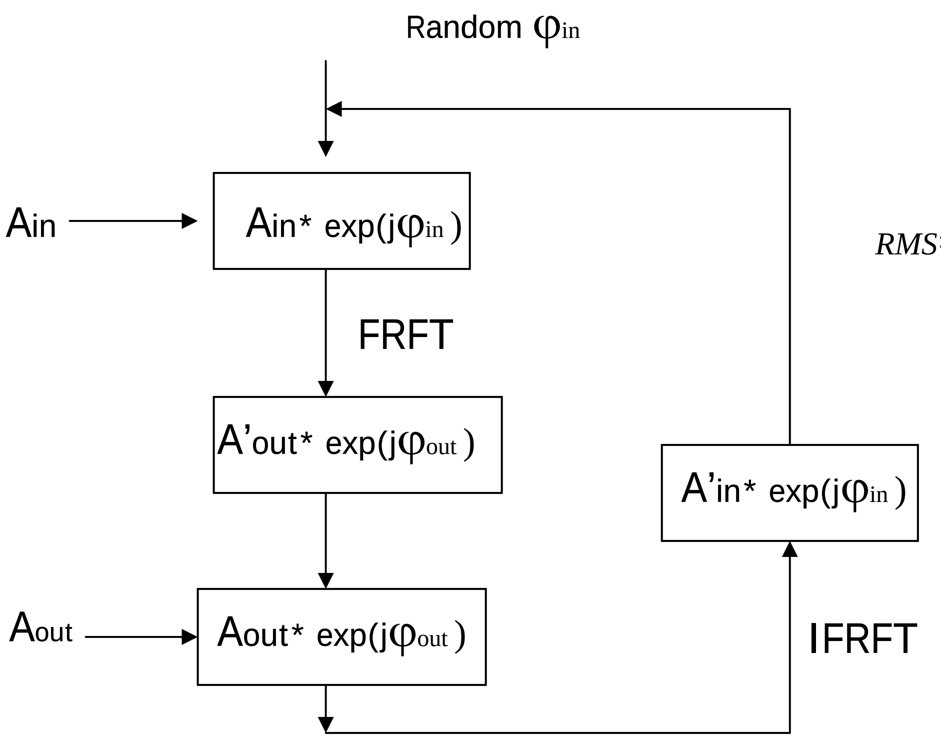
The flowchart of Error reduction algorithm(double-intensity measurements)



$$RMS=\left[\int_{-\infty}^{+\infty}\left(Aout-\left|F^{q}\{Ain\times\exp(j\Phi in)\}\right|\right)^{2}dx\right]^{1/2}$$

2. Fractional Fourier Transform

1) Definition

$$G(X) = \{F^q g\}X = \int_{-\infty}^{\infty} g(x) K_q(x, X) dx$$

$$K_q(x, X) = A_\Phi \exp\{j\pi[(x^2 + X^2)\cot\Phi - 2Xx\csc\Phi]\}$$

$$A_\Phi = \frac{\exp[-j\pi\,\mathrm{sgn}(\sin\Phi)/4 + j\Phi/2]}{\sqrt{|\sin\Phi|}}$$

where $\Phi = q\pi/2$, and q is the fractional order of FRFT

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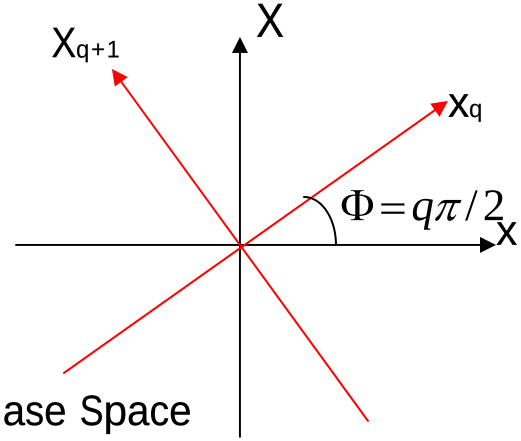
2) Properties

a) Linearity;

b) F^1 is the ordinary Fourier transform;

c) Index additivity $F^{q1} F^{q2} = F^{q1+q2}$

3) Relationship between Wigner distribution and FRFT



FRFT G(X) is merely a rotated version of the Wigner distribution of g(x), i.e.

$$G(X)=W_g(x\cos(\Phi)-X\sin(\Phi),\ x\sin(\Phi)+X\cos(\Phi))$$

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4) Relationship between Fresnel diffraction and FRFT

The FRFT is the mathematical representation of Fresnel diffraction.
(FT is the mathematical representation of Fraunhoffer diffraction)

The continuity of FRFTs from the zero order to the first order correspond to the no diffraction to Fraunhoffer diffraction passing by Fresnel diffraction.

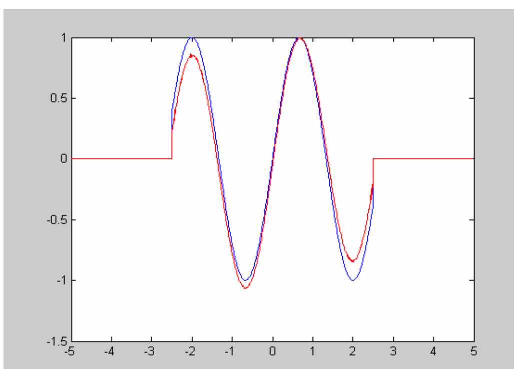
3. Problem to be solved for error reduction algorithm

We need to find out an exact fast algorithm for the fractional Fourier transform same as FFT

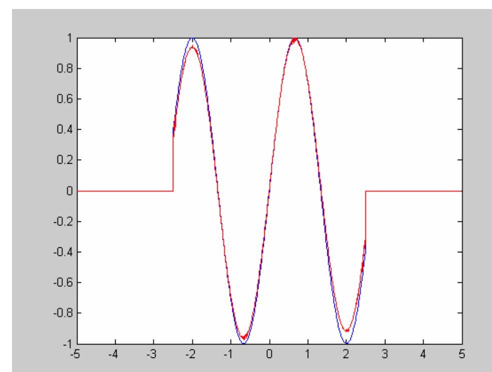
4. Preliminary results

1) one dimensional signal

Fractional order 0.4 iterations3000



Fractional order 0.6 iterations1358



Orginal signal y=exp(j*0.75*pi*sin(x))