Applications and methods of Wavefront measurement

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Adaptive Optics

- Adaptive = feedback control
- Adaptive Optics
  - 3 Components
    - Wavefront Modulator (WFM)
    - Wavefront Sensor (WFS)
    - Control loop
- Active optics = no feedback
  - No WFS
  - No on-line control loop
  - Control signal pre-computed off-line (e.g. gravity sag, ...)
Example

Retinal image corrected for aberrations of anterior optics of the eye (Univ of Rochester)
Example

Image from CFHT at J band (1.65\textmu m)
Image Quality

- AO correction increases Strehl but residual errors still have $r_0$ scale

- AO ‘corrected’ images have ‘core’ and ‘skirt’
Strehl ratio

- $S \sim \exp(-\sigma^2_\phi)$
- Image peak brightness falls rapidly with D/r₀
- Small errors
  - (λ/4 or less) > good images, S>0.4
  - λ/10 or less S>0.7
So…

- For imaging $\pm \lambda/10$ correction is very good
- For spectroscopy this is OK except for crowded-field work
- Is AO correction to $< \lambda/10$ practical?  
  - Probably only in rare circumstances...

- What about non-astronomical, non-imaging applications?
Optical metrology

• Metrology of optical wavefronts can give:
  – Surface shape
  – Positional information
  – Depth information

• This non-contact method can:
  – be used at any $\lambda$
  – give high accuracy (best results $\pm 0.7\text{nm}$) and be time resolved
Other Applications

- Monitoring processes - laser-welding, laser drilling, fluid flow, ...
- Material inhomogeneity
- Optical components and assembly testing
- Robotic imaging

- shape of weld, beam control, turbulence measurement, ...
- tomographic measurement
- non-interferometric tolerancing, validation
- 3-d scene
WFS Requirements

• For metrology applications high accuracy is required:
  ➢ Request for $\lambda/1000$ in float glass industry
  ➢ Request for $\lambda/40000$ in telecomms!!

• Depth measurement to $\sim 1\mu m$ in biomedical applications
Thin-film induced wavefront aberrations

- The Fresnel reflection from the rear surface of a thin film provides
  - displaced image of source (tilt, defocus)
  - spherical aberration of source image
  - other aberrations

Film thickness from 100nm to 10μm

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Wavefront sensors

Technical basis

- Wavefront slope
- Wavefront curvature
- Image quality criteria

Techniques

- Shack-Hartmann
- Shearing interferometer
- Wavefront curvature sensor
- Phase-diversity wavefront sensor
- e.g. max of integral of intensity squared
Shack-Hartmann Wavefront Sensor

- Shack-Hartman WFS used in most AO applications
- Wavefront reconstructed from integration of local tilts
- Regions over which tilts are measured are defined by lenslet matrix
Shack-Hartmann Wavefront Sensor

- Anecdotal evidence suggests that calibration is a significant problem
  - may be solved with chip-scale SH-WFS

- Best reported measurements ~ $\lambda/100$ defocus error measurement (Wavefront Sciences, July 2001) - unpublished to date
Intensity Transport Equation

- Parabolic wave eqn
  \[
  \left( i \frac{\partial}{\partial z} + \frac{\nabla^2}{2k} + k \right) u_z(r) = 0
  \]

- Let
  \[
  u_z(r) = \sqrt{I_z(r)} \exp(i\phi_z(r))
  \]

- Multiply PWE by \( u^* \) on the left and by \( u \) on the right - take the difference and...
  \[
  -k \frac{\partial}{\partial z} I_z(r) = \nabla \cdot (I_z(r) \nabla \phi_z(r))
  \]
ITE solution

- Expanding ITE containing…
  - A curvature term
  - A slope term

- If intensity is const

- ITE becomes

\[-k \frac{\partial}{\partial z} I_z(r) = I_z(r)\nabla^2 \phi_z(r) + \nabla I_z(r) \cdot \nabla \phi_z(r)\]

\[\nabla I_z(r) = 0\]

\[-\frac{k}{I_z(r)} \frac{\partial}{\partial z} I_z(r) = \nabla^2 \phi_z(r)\]
Phase-diverse wavefront sensing
(wavefront curvature sensing)

- Solution of ITE gives wavefront

\[ \Psi(r) = -k \int_{R} dr' G(r, r') \frac{\partial I(r')}{\partial z} \]

\[ \frac{I_{\text{Plane 1}} - I_{\text{Plane 2}}}{z_1 - z_2} \sim \frac{\partial I}{\partial z} \]
Why Phase Diversity?

• Phase-diversity can operate in the far-field pupil space (c.f. aperture synthesis)
  – Source structure is encoded in correlations of wavefront, not in wavefront itself

• Algorithm well-known
  – Previously implemented as an iterative procedure
Synthesis Imaging

- An array of holes acts like a large, masked lens
- Radio astronomy methods unsuited to snapshot use
- Redundant Spacings Calibration (RSC) > ‘snapshot’ use
- Redundancy is a required for unique data inversion
Redundancy in Synthesis Imaging

- N apertures \( \Rightarrow \) N(N-1)/2 Fourier components
- Unknown phase for each aperture
- \# data < \# unknowns \( \Rightarrow \) parametric solutions
- Solve through the use of redundancy (e.g. CLEAN)

- Ways to get redundancy:
  - model-building
  - constraint object support \( \Rightarrow \) Fourier interpolation
  - redundant observations (RSC)
- Far-field/pupil-plane \( \Rightarrow \) source structural information delocalised
Phase diversity/wavefront curvature

\[
\frac{2\pi (I_1 - I_2)}{\lambda \int \delta z} \cdot G = \phi
\]
How to collect data?

IMP® gratings

C.f. twin images in holography
Diffractive Optics

- Phase-diversity scheme needs wavefront intensity pattern on two separate planes: Scheme adopted uses IMP®'s

**Undistorted Grating** - identical images of a single object layer in each order

**Distorted Grating** - images of different object layers on a single image plane

IMP® is a DERA Trademark

DERA is now QinetiQ

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Experimental Validation

• Test wavefronts
  – Pure Zernike modes
  – Mixture of Zernike modes
  – Random wavefront errors

• Experimental 3-d imaging
  – Layers imaged as close together as 50 μm
  – Layers imaged typically several metres apart
  – In principle, layers can be kilometres apart
  – 9 layers imaged experimentally
  – Up to 25 Layer imaging designed

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Applications of optical metrology

- Surface profiling
- Monitoring processes - welding, laser drilling, fluid flow, ...
- Material inhomogeneity
- Position sensing
- Robotic imaging

- λ/1000 accuracy?
- Depth of hole, shape of weld, turbulence measurement
- Tomographic reconstruction
- Bearing and (short) distances
- 3-d information about scene
Applications of Wavefront Sensing

- Component of AO system
- Testing optical components and complete assemblies
- Laser-beam quality, $M^2$

- Modal or zonal feedback to wavefront modulator
- Non-interferometric quality control, esp at non-visible wavelengths
- Measure spot on several planes
Phase-diverse wavefront sensing

- Measured curvature vs set curvature shows systematic trends
- Possible errors in
  - alignment
  - cross-talk
    were eliminated
    (effect was too large)

\[ y = 0.0278x^2 + 0.8805x + 0.0041 \]

Deviation from straight line ~ $\lambda/100$
Deviation from quadratic ~ $\lambda/300$

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How to test the WFS at $\lambda/1000$?

- Can we generate a test wavefront with $\lambda/1000$ precision?

A point source at $z$ gives known wavefront curvature

$z$ is the set distance to the source

$\Delta z$ is the accuracy with which $z$ can be set

$\Delta s$ is the maximum error in set wavefront curvature

$r$ is the pupil radius

$s \sim -\frac{r^2}{2z} \Rightarrow z \geq \sqrt{\frac{r^2}{2} \frac{\Delta z}{\Delta s}}$

If $r = \sqrt{2} \times 10^{-2}; \Delta s = 5 \times 10^{-10}; \Delta z = 5 \times 10^{-3}$

$z > 30$m
How to test the WFS at $\lambda/1000$?

- At 30m bench is not widely available
- A folded path is difficult to measure

- Fold the optical path
- Absolute validation difficult by this route
- Thus use relative curvature induced by small displacements of the source
How to test the WFS at $\lambda/1000$?

- How do we set the lab tests at finite distance?
- Model as shown below with source focussed on camera
How to test the WFS at $\lambda/1000$?

• Combination focal length

$$f_0 = \frac{f_1 f_2}{f_1 + f_2}$$

if $f_1 = z; f_2 = v$

- satisfies usual lens eqns
- input collimated between lenses
- normal wfs description if grating between lenses

$z \rightarrow z + \Delta z$

• What if source is shifted without re-focussing?
What is wavefront ‘sag’ between lenses if source is shifted?

- Image distance gives virtual source position

\[ \nu_{\Delta z} = \frac{f_0(z + \Delta z)}{z + \Delta z - f_0} \implies z + \frac{z^2}{\Delta z} \]

- ‘Sag’ is given by

\[ s = -\frac{r^2}{2z(1 + \frac{z}{\Delta z})} \]

\[ \sim -\frac{r^2}{2z} \times \frac{\Delta z}{z} \times \left(1 - \frac{\Delta z}{z} + \ldots \right) \]

\[ \sim -\frac{r^2\Delta z}{2z^2} + \frac{r^2(\Delta z)^2}{2z^3} + \ldots \]

- So expect to see linear + quadratic behaviour
Measurements

\[ y = -0.8979x \]

\[ y = -0.0324x^2 - 0.8979x - 0.0011 \]

\[ y = -0.0278x^2 - 0.8805x - 0.0041 \]

Exp Defocus
Theo Defocus
Linear (.)

\[ y = -0.8979x \]

\[ y = -0.0278x^2 - 0.8805x - 0.0041 \]

\[ y = -0.0324x^2 - 0.8979x - 0.0011 \]
Setting the curvature

Images in diffraction orders have same scale iff the source is imaged in central order.

Change curvature to test and calibrate measurements
Phase-diverse wavefront sensing

Requirements

- Need Green’s function
- Need to get boundary conditions correct
- Need accurate intensity gradient

- Calculated at DERA*
- Get from $I_1 - I_2$ at pupil edges and $z_1 - z_2$
- Need accurate spacing to get accurate gradient

*now QinetiQ
Phase-diverse Wavefront Sensing

Effects of wavefront shape

a) defocus
b) astigmatism
c) coma
d) trefoil
e) spherical aberration

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How critical are placements?

Provided that source is not in the near field and the IMP® is in front of the lens, the positions of the planes are reliably defined.

\[ U_0 = -5 \text{ m} \text{Red is } U_{+1} \text{ and Green is } U_{-1} \]
Best Results

Noise on Curvature with a 150 mm lens

Samples

λ/167
λ/200
λ/250
λ/333
λ/500
λ/1000
What of remaining effects?

- Background subtraction
  - Vital for accuracy
  - Darkened lab but computer screens in lab vary in brightness
  - No spectral filters used (avoid unknown aberrations)

- No attempt in lab to control temperature
- No attempt in lab to control air currents
- Jitter on translation stage
- Numerical output to only 3 decimal places
Restrictions

- Intensity assumed uniform
- Wavefronts continuous with continuous 1st derivative
- Close measurement plane to approx derivative
- Metrology implies laser illumination, implies speckle
- Disadvantage if measuring man-made surfaces
- Dynamic range is restricted
Wavefront Sensing Schematic
Other diversity Kernels?

- Why restrictions?
- But ITE is an approximation
- What is special about phase diversity/wavefront curvature?

▶ Solution constraints on ITE
▶ Iterative solutions reconstruct discontinuities
▶ Easy interpretation
▶ Local measurement
▶ Fast implementation

Greater flexibility is likely if the approximation of the ITE is avoided and other kernels investigated for WFS - e.g. RSC
Anisoplanatism

By similar triangles

\[
\frac{d}{R_1} = \frac{D}{R_2} \implies \frac{R_2}{R_1} = \frac{D}{d}
\]

terrestrial imaging is almost always severely anisoplanatic
Conclusions

- Phase-diversity WFS based on IMP® technology is capable of \( \lambda/1000 \) accuracy
- Accuracy is \( \lambda \) independent, best 0.7nm
- Control of background subtraction is greatest problem with present arrangement
- Greater flexibility likely using more complete description and other diversity kernels