Applications and methods of Wavefront measurement

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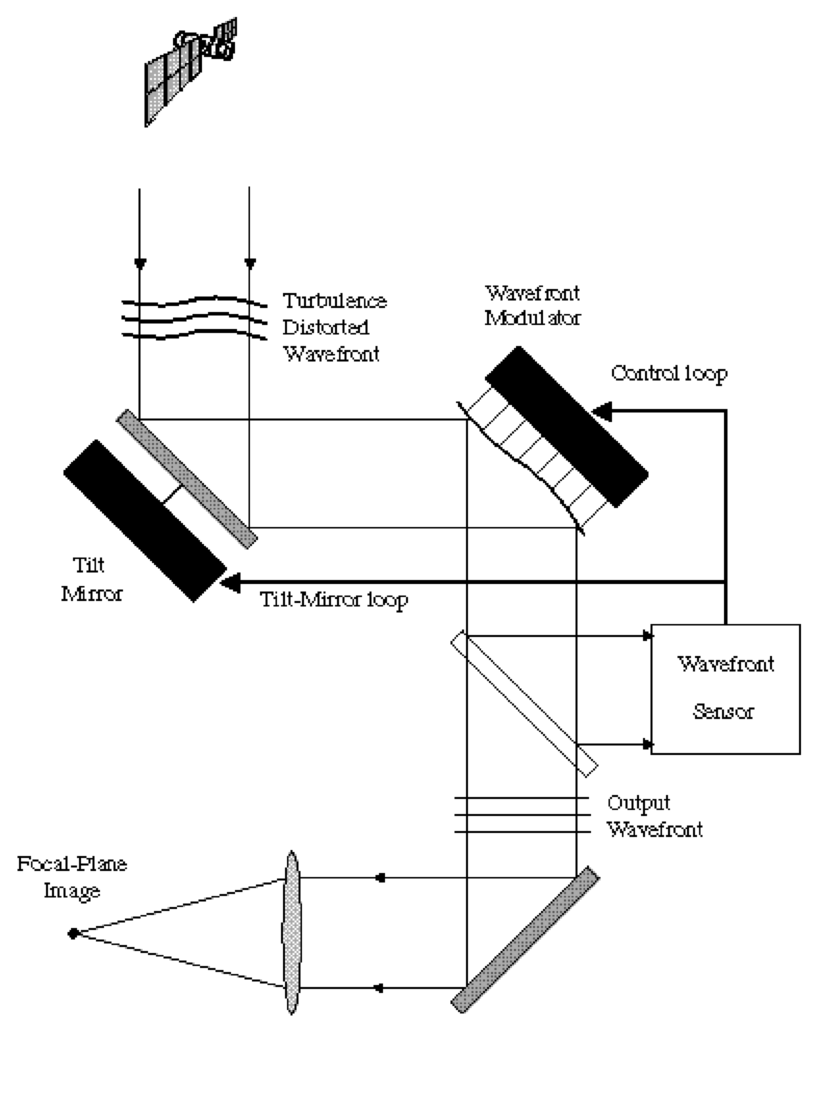
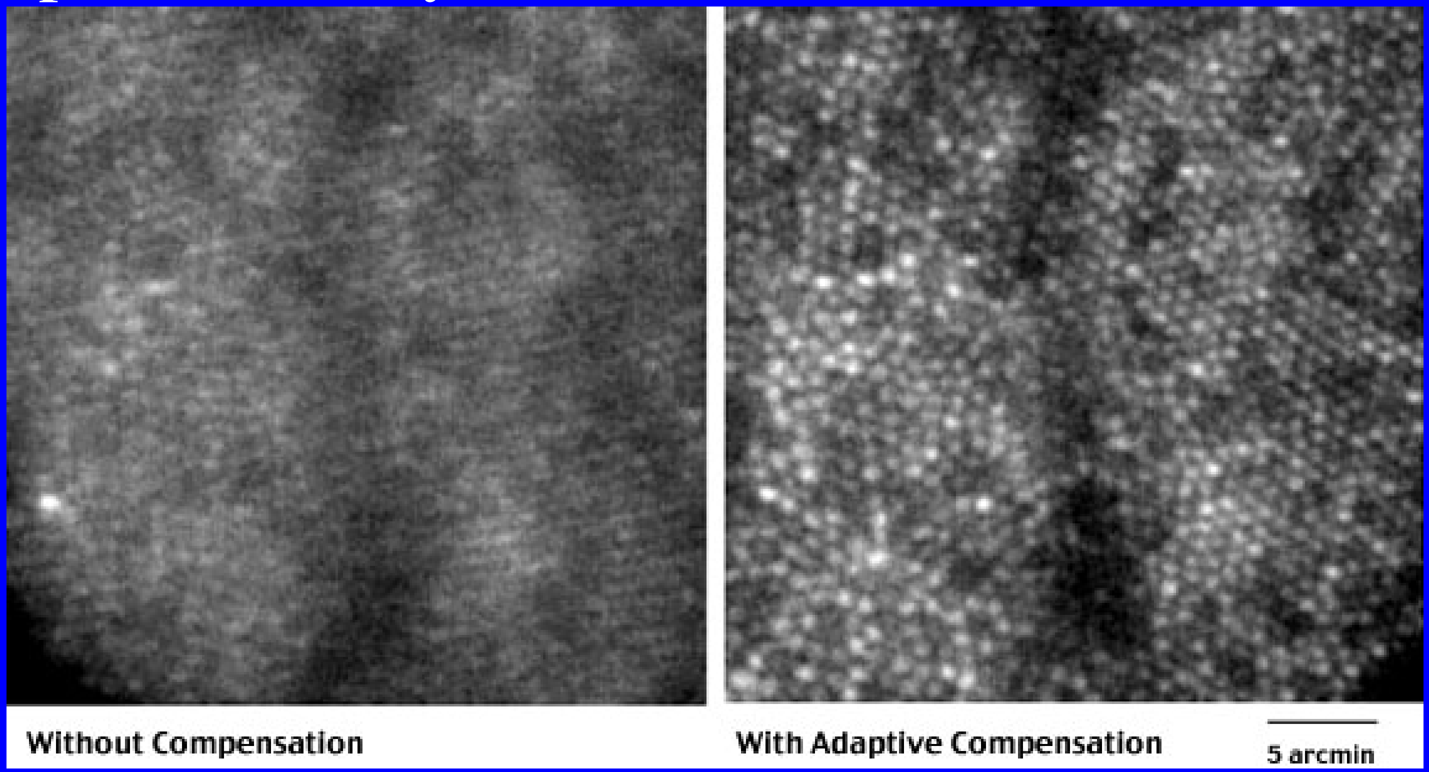
Adaptive Optics



- **Adaptive = feedback control**
- Adaptive Optics
  - 3 Components
    - Wavefront Modulator (WFM)
    - Wavefront Sensor (WFS)
    - Control loop
- Active optics = no feedback
  - No WFS
  - No on-line control loop
  - Control signal pre-computed off-line (e.g. gravity sag, ...)

Example

Retinal image corrected for aberrations of anterior optics of the eye (Univ of Rochester)



Example

Image from CFHT at J band (1.65μm)

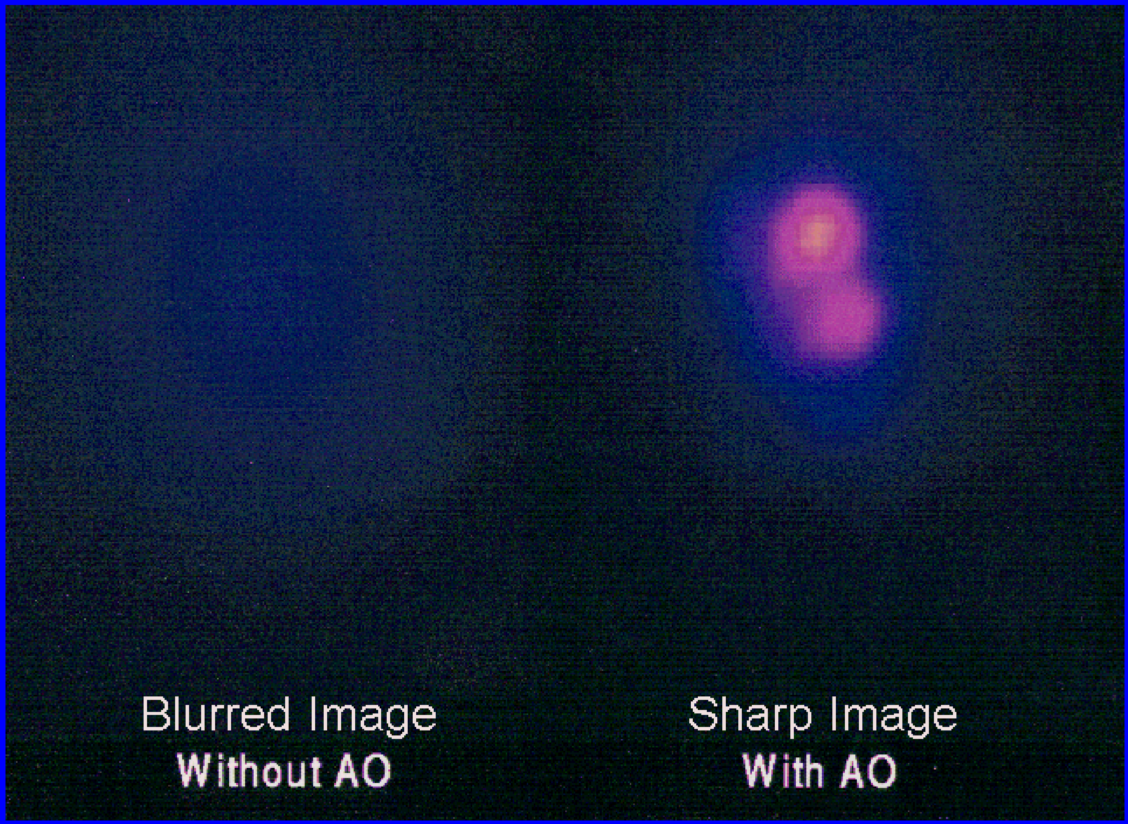
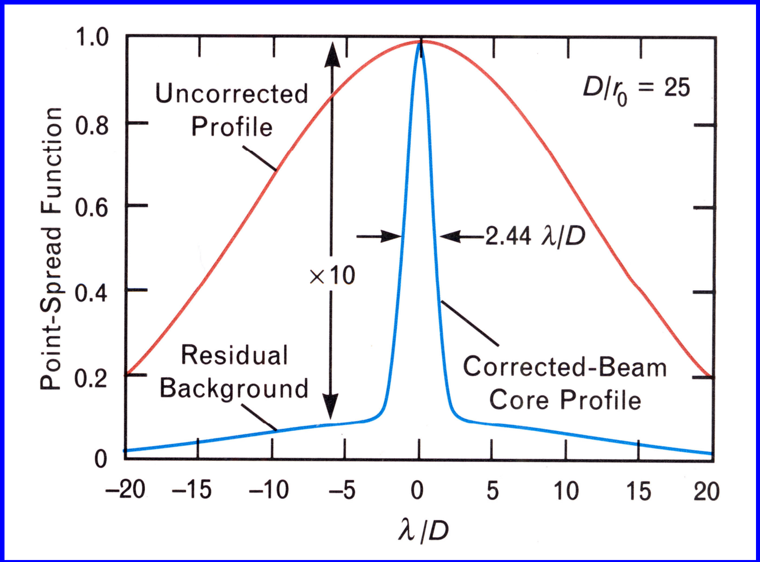


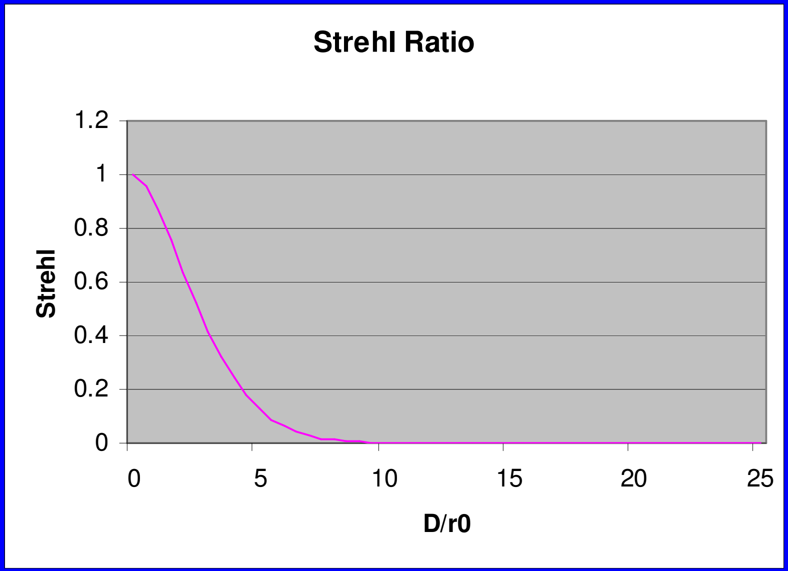
Image Quality



- AO correction increases Strehl but residual errors still have r_0 scale
- AO 'corrected' images have 'core' and 'skirt'

Strehl ratio

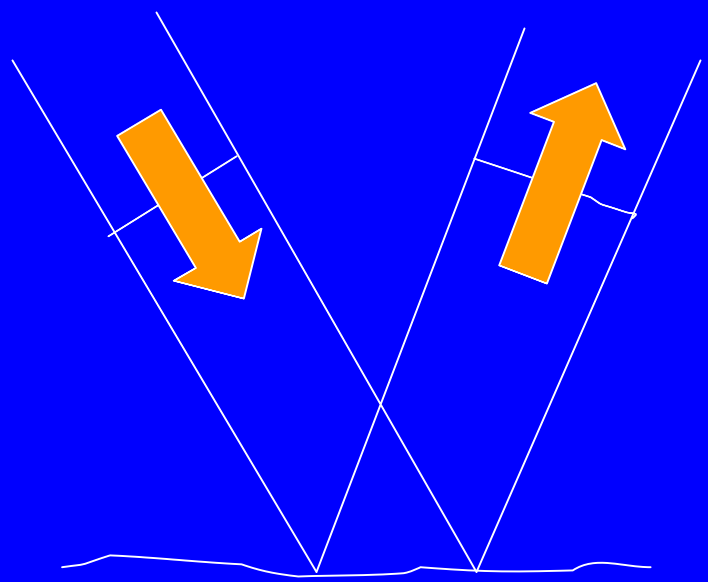
- $S \sim \exp\left(-\sigma_\phi^2\right)$
- Image peak brightness falls rapidly with D/r_0
- Small errors
  - (λ/4 or less) > good images, S>0.4
  - λ/10 or less S>0.7



So…

- For imaging $\pm\lambda/10$ correction is very good
- For spectroscopy this is OK except for crowded-field work
- Is AO correction to $<\lambda/10$ practical?
  – Probably only in rare circumstances...
- What about non-astronomical, non-imaging applications?

Optical metrology



- Metrology of optical wavefronts can give:
  - Surface shape
  - Positional information
  - Depth information
- This non-contact method can:
  - be used at any λ
  - give high accuracy (best results $\pm$0.7nm) and be time resolved

Other Applications

• Monitoring processes - laser-welding, laser drilling, fluid flow, ...	➢ shape of weld, beam control, turbulence measurement, …
• Material inhomogeneity	➢ tomographic measurement
• Optical components and assembly testing	➢ non-interferometric tolerancing, validation
• Robotic imaging	➢ 3-d scene

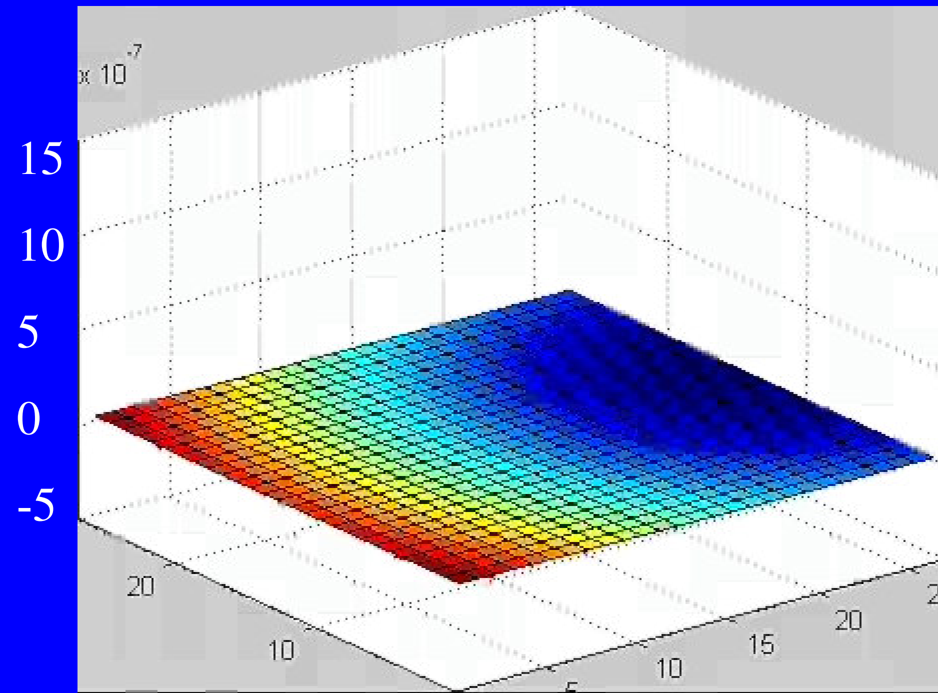
WFS Requirements

- For metrology applications high accuracy is required:
  - Request for $\lambda/1000$ in float glass industry
  - Request for $\lambda/40000$ in telecomms!!

- Depth measurement to ~ 1μm in bio-medical applications

Thin-film induced wavefront aberrations

- The Fresnel reflection from the rear surface of a thin film provides
  - displaced image of source (tilt, defocus)
  - spherical aberration of source image
  - other aberrations

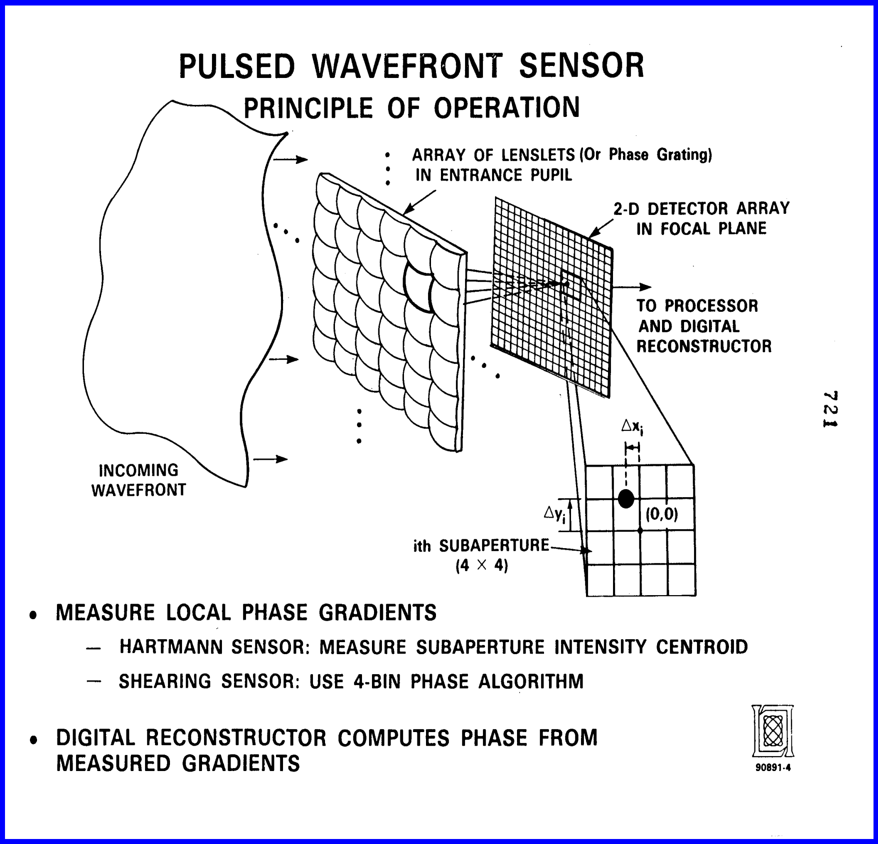


Film thickness from 100nm to 10µm

Wavefront sensors

Technical basis	Techniques
◆ Wavefront slope	➢ Shack-Hartmann ➢ Shearing interferometer
◆ Wavefront curvature	➢ Wavefront curvature sensor ➢ Phase-diversity wavefront sensor
◆ Image quality criteria	➢ e.g. max of integral of intensity squared

Shack-Hartmann Wavefront Sensor



- Shack-Hartman WFS used in most AO applications
- Wavefront reconstructed from integration of local tilts
- Regions over which tilts are measured are defined by lenslet matrix

Shack-Hartmann Wavefront Sensor

- Anecdotal evidence suggests that calibration is a significant problem
  - may be solved with chip-scale SH-WFS
- Best reported measurements ~ $\lambda/100$ defocus error measurement (Wavefront Sciences, July 2001) - unpublished to date

Intensity Transport Equation

- Parabolic wave eqn $$\left(i\frac{\partial}{\partial z}+\frac{\nabla^2}{2k}+k\right)u_z(r)=0$$
- Let $$u_z(r)=\sqrt{I_z(r)}\exp\left(i\phi_z(r)\right)$$
- Multiply PWE by u^* on the the left and by u on the right - take the difference and...

$$-k\frac{\partial}{\partial z}I_z(r)=\nabla.(I_z(r)\ \nabla\phi_z(r))$$

ITE solution

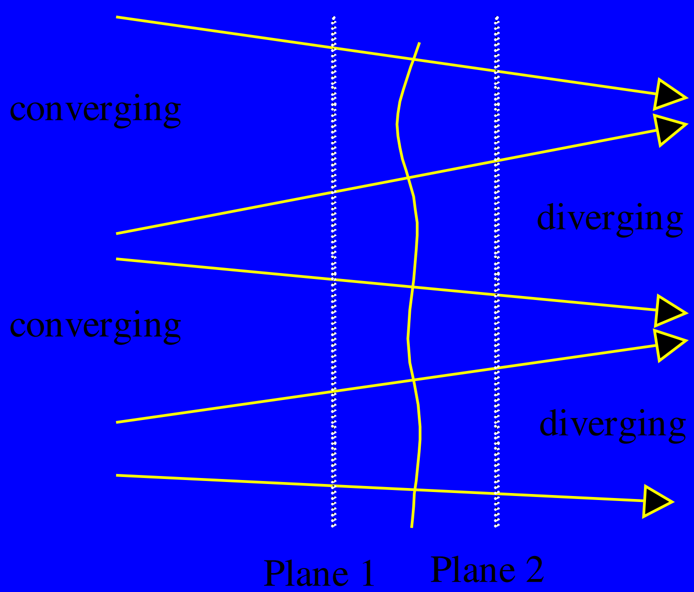
- Expanding ITE containing…. $$-k\frac{\partial}{\partial z}I_z(r)=I_z(r)\nabla^2\phi_z(r)+\nabla I_z(r).\nabla\phi_z(r)$$
  - A curvature term $I_z(r)\nabla^2\phi_z(r)$
  - A slope term $\nabla I_z(r).\nabla\phi_z(r)$

- If intensity is const $\nabla I_z(r)=0$

- ITE becomes $$-\frac{k}{I_z(r)}\frac{\partial}{\partial z}I_z(r)=\nabla^2\phi_z(r)$$

Phase-diverse wavefront sensing

(wavefront curvature sensing)



- Solution of ITE gives wavefront

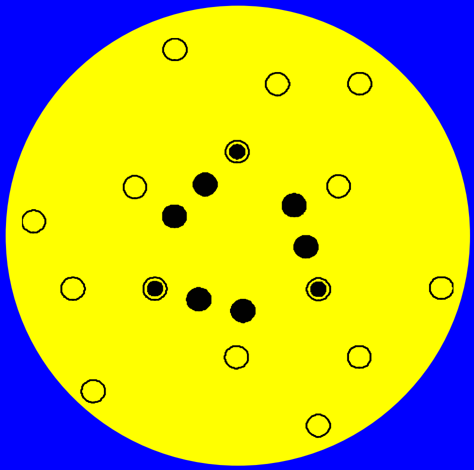
$$\Psi(r) = -k \int_R dr' G(r, r') \frac{\partial I(r')}{\partial z}$$

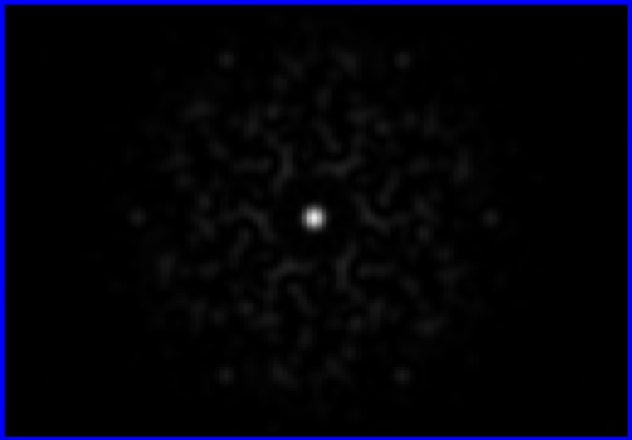
$$\frac{I_{\text{Plane 1}} - I_{\text{Plane 2}}}{z_1 - z_2} \sim \frac{\partial I}{\partial z}$$

Why Phase Diversity?

- Phase-diversity can operate in the far-field pupil space (c.f. aperture synthesis)
  - Source structure is encoded in correlations of wavefront, not in wavefront itself
- Algorithm well-known
  - Previously implemented as an iterative procedure

Synthesis Imaging





- An array of holes acts like a large, masked lens
- Radio astronomy methods unsuited to snapshot use
- Redundant Spacings Calibration (RSC) > ‘snapshot’ use
- Redundancy is a required for unique data inversion

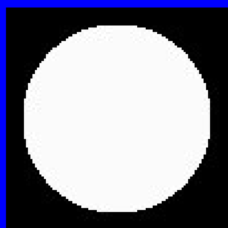
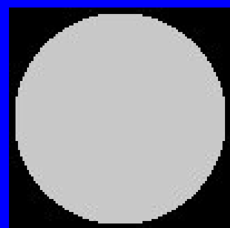
Redundancy in Synthesis Imaging

- N apertures ➢ N(N-1)/2 Fourier components
- Unknown phase for each aperture
- # data < # unknowns
  ➢ parametric solutions
- Solve through the use of redundancy (e.g. CLEAN)

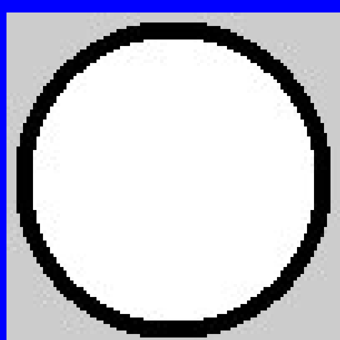
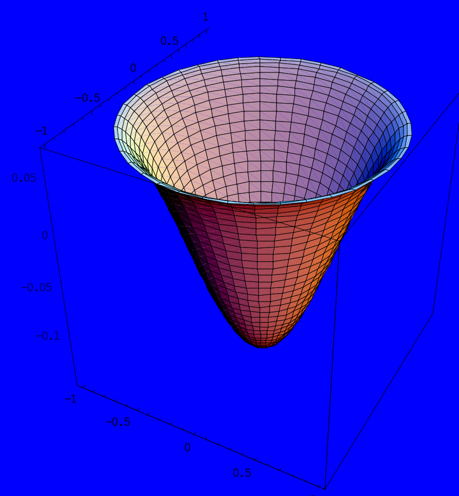
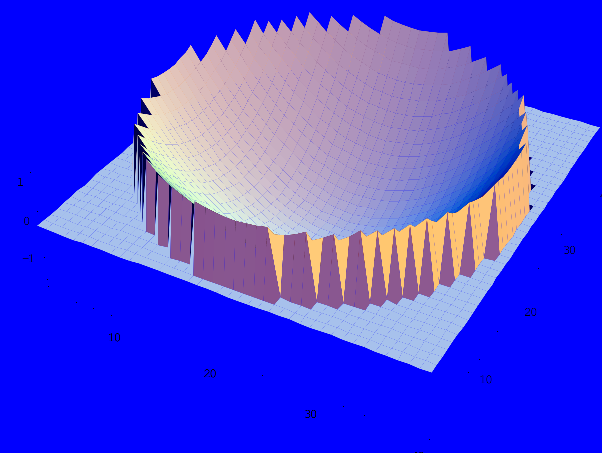
- Ways to get redundancy:
  - model-building
  - constraint object support
    ➢ Fourier interpolation
  - redundant observations (RSC)
- Far-field/pupil-plane
  ➢source structural information delocalised

Phase diversity/wavefront curvature

I_1 I_2

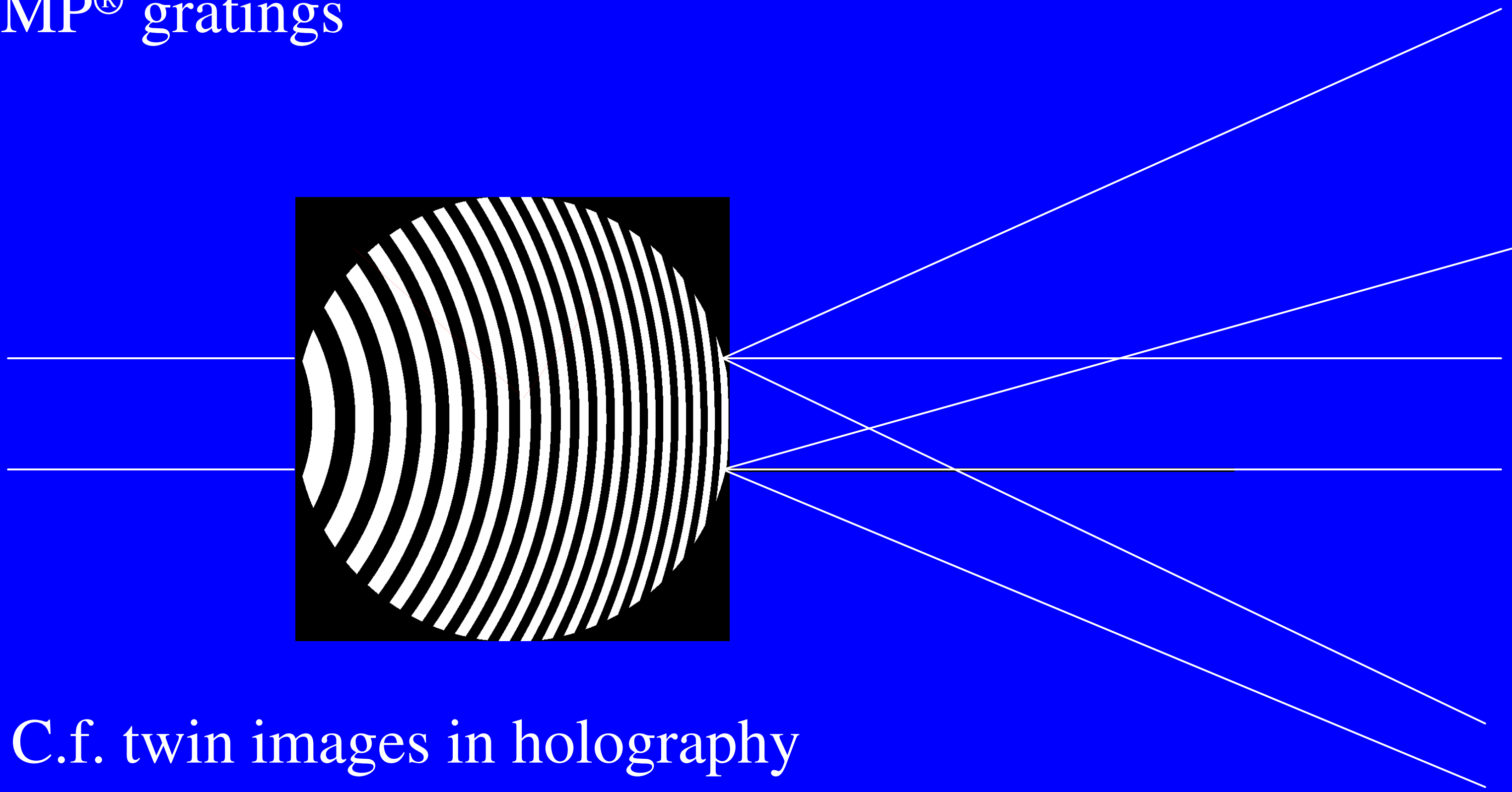
 - 

$$\frac{2\pi}{\lambda}\cdot\frac{(I_1-I_2)}{\bar{I}\,\delta z}\cdot G=\phi$$

 •  = 

How to collect data?

IMP® gratings

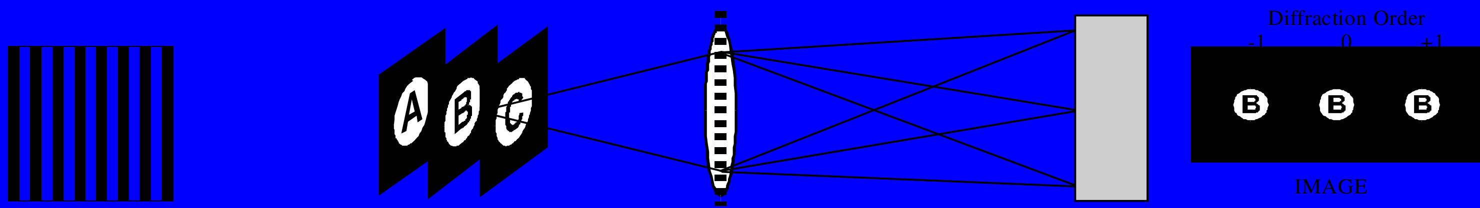


C.f. twin images in holography

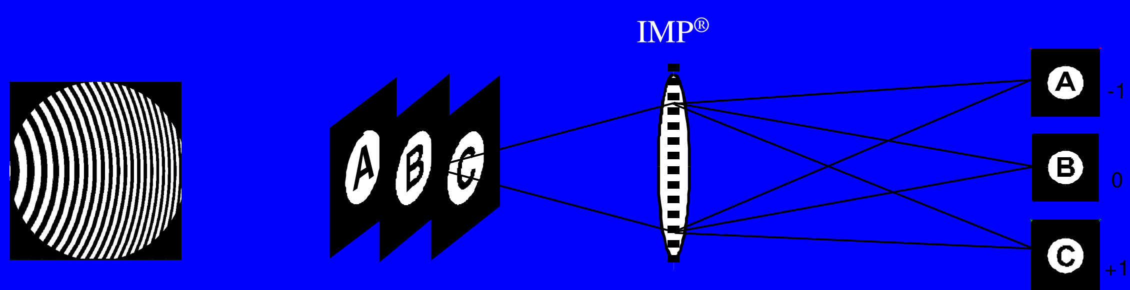
Diffractive Optics

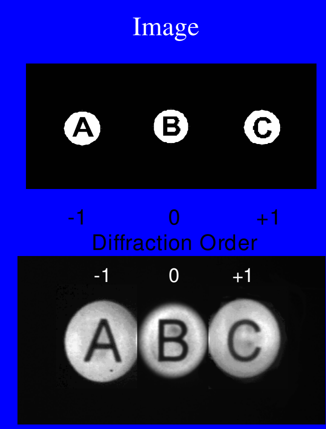
- Phase-diversity scheme needs wavefront intensity pattern on two separate planes: Scheme adopted uses IMP®s

Undistorted Grating - identical images of a single object layer in each order



Distorted Grating - images of different object layers on a single image plane





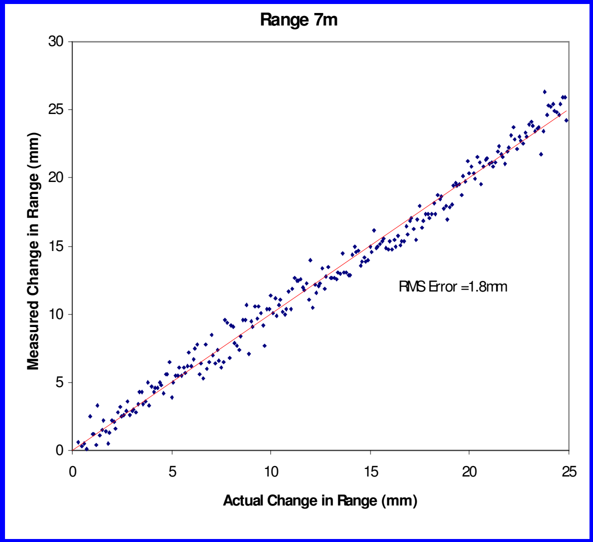
IMP® is a DERA Trademark

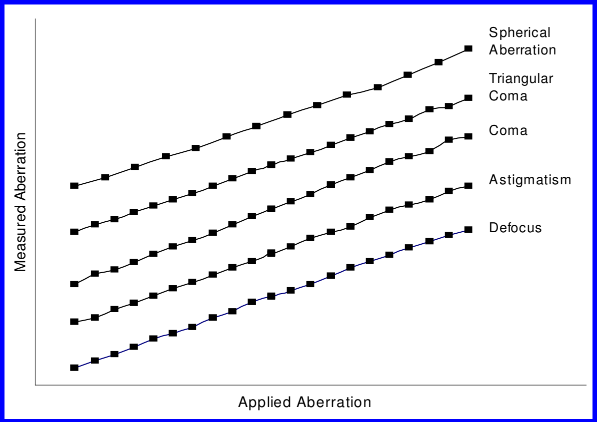
DERA is now QinetiQ

Experimental Validation

- Test wavefronts
  - Pure Zernike modes
  - Mixture of Zernike modes
  - Random wavefront errors

- Experimental 3-d imaging
  - Layers imaged as close together as 50 μm
  - Layers imaged typically several metres apart
  - In principle, layers can be kilometres apart
  - 9 layers imaged experimentally
  - Up to 25 Layer imaging designed





Applications of optical metrology

- Surface profiling
  - $\lambda/1000$ accuracy?
- Monitoring processes - welding, laser drilling, fluid flow, ...
  - Depth of hole, shape of weld, turbulence measurement
- Material inhomogeneity
  - Tomographic reconstruction
- Position sensing
  - Bearing and (short) distances
- Robotic imaging
  - 3-d information about scene

Applications of Wavefront Sensing

- Component of AO system
- Testing optical components and complete assemblies
- Laser-beam quality, M^2

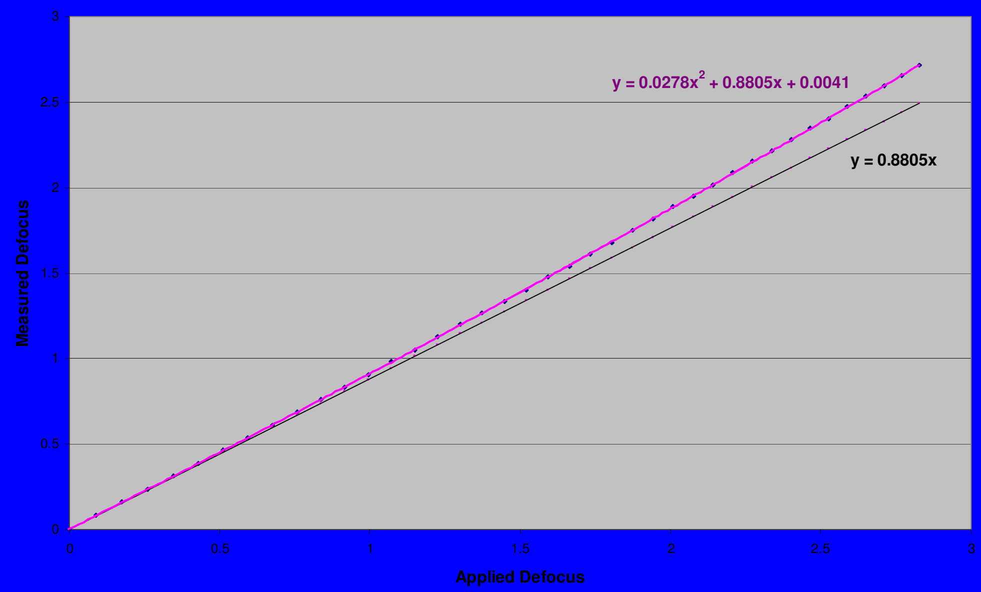
- Modal or zonal feedback to wavefront modulator
- Non-interferometric quality control, esp at non-visible wavelengths
- Measure spot on several planes

Phase-diverse wavefront sensing

- Measured curvature vs set curvature shows systematic trends

- Possible errors in
  - alignment
  - cross-talk

  were eliminated (effect was too large)



Deviation from straight line ~ $\lambda/100$

Deviation from quadratic ~$\lambda/300$

How to test the WFS at λ/1000?

- Can we generate a test wavefront with λ/1000 precision?
  - ➢ A point source at z gives known wavefront curvature

z is the set distance to the source

Δz is the accuracy with which z can be set

Δs is the maximum error in set wavefront curvature

r is the pupil radius

$$s \sim -\frac{r^2}{2z} \Rightarrow z \geq \sqrt{\frac{r^2}{2}\frac{\Delta z}{\Delta s}}$$

If $r = \sqrt{2}\times 10^{-2}; \Delta s = 5\times 10^{-10}; \Delta z = 5\times 10^{-3}$

➢ z > 30m

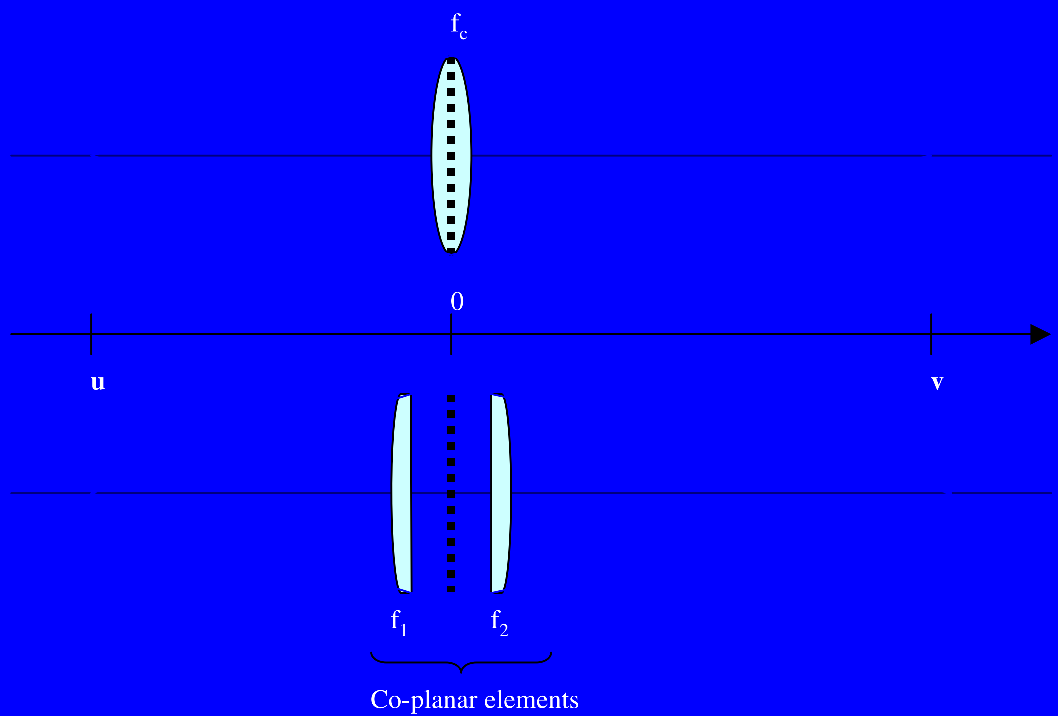
How to test the WFS at $\lambda/1000$?

- At 30m bench is not widely available
- A folded path is difficult to measure

- Fold the optical path
- Absolute validation difficult by this route
- Thus use relative curvature induced by small displacements of the source

How to test the WFS at λ/1000?

- How do we set the lab tests at finite distance?
- Model as shown below with source focussed on camera



How to test the WFS at λ/1000?

- Combination focal length $f_0 = \dfrac{f_1 f_2}{f_1 + f_2}$

if $f_1 = z;\ f_2 = v$

- ➢satisfies usual lens eqns
- ➢input collimated between lenses
- ➢normal wfs description if grating between lenses

$z \to z + \Delta z$

- What if source is shifted without re-focussing?

What is wavefront 'sag' between lenses if source is shifted?

- Image distance gives virtual source position

$$v_{\Delta z} = \frac{f_0(z+\Delta z)}{z+\Delta z - f_0} \overset{f_0=z}{\Longrightarrow} z + \frac{z^2}{\Delta z}$$

- 'Sag' is given by

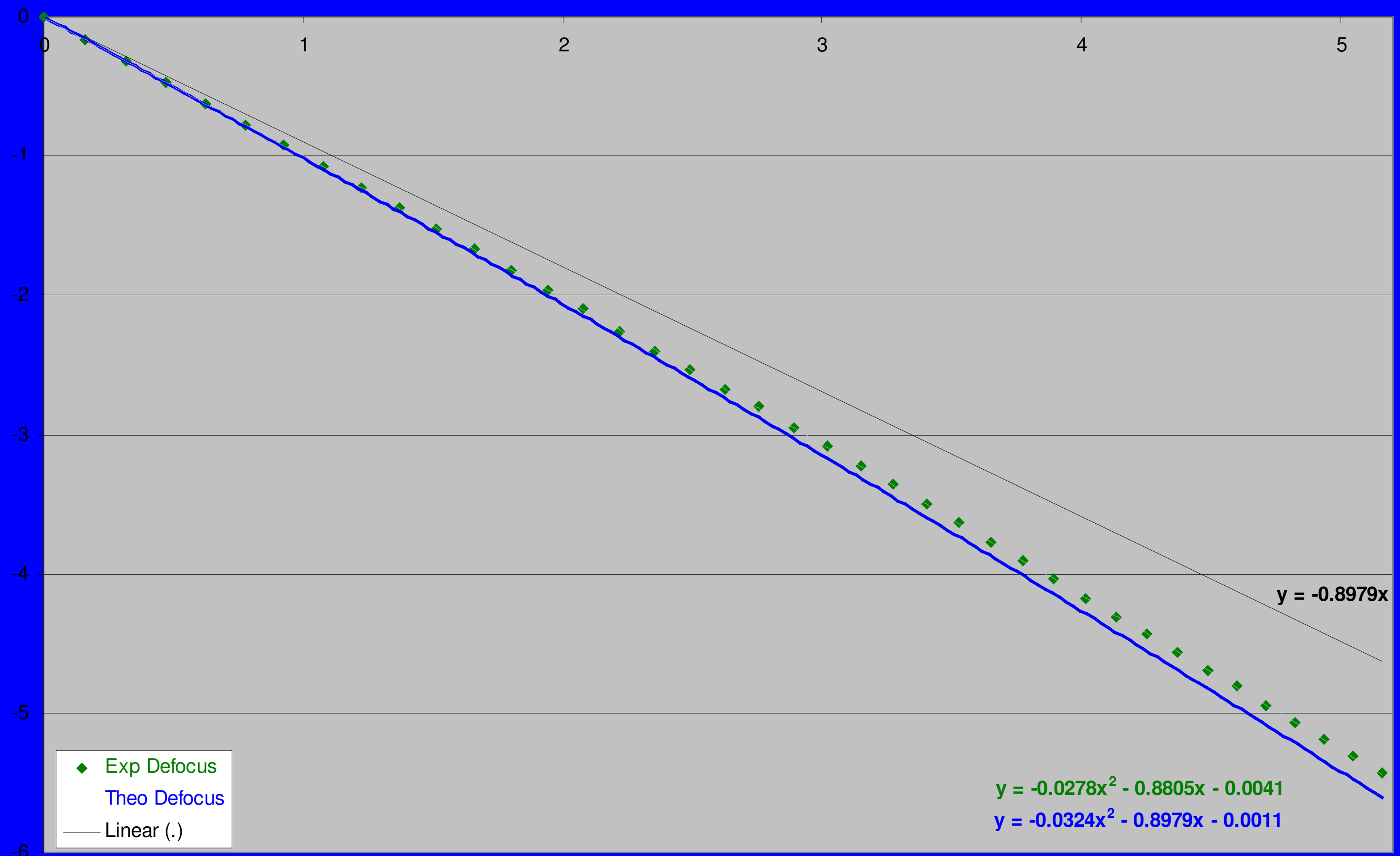
$$s = -\frac{r^2}{2z(1+\frac{z}{\Delta z})}$$

$$\sim -\frac{r^2}{2z} \times \frac{\Delta z}{z} \times \left(1 - \frac{\Delta z}{z} + \ldots\right)$$

$$\sim -\frac{r^2 \Delta z}{2z^2} + \frac{r^2 (\Delta z)^2}{2z^3} - \ldots$$

- So expect to see linear + quadratic behaviour

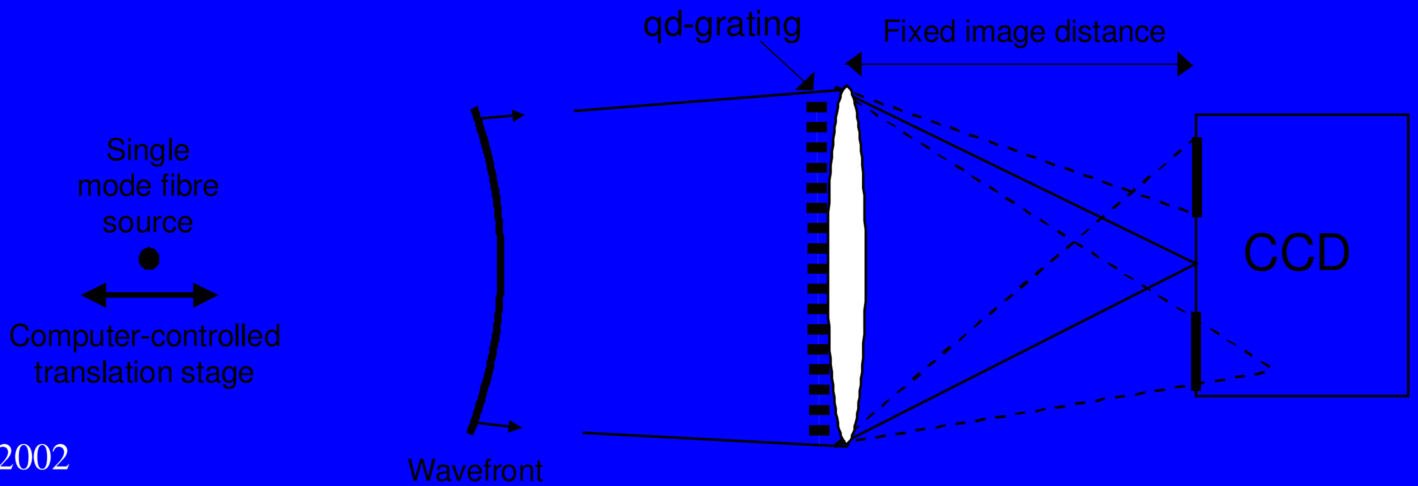
Measurements



Setting the curvature

Images in diffraction orders have same scale iff the source is imaged in central order.

Change curvature to test and calibrate measurements



Phase-diverse wavefront sensing

Requirements

- Need Green's function
- Need to get boundary conditions correct
- Need accurate intensity gradient

- Calculated at DERA*
- Get from I_1-I_2 at pupil edges and z_1-z_2
- Need accurate spacing to get accurate gradient

*now QinetiQ

Phase-diverse Wavefront Sensing

Effects of wavefront shape

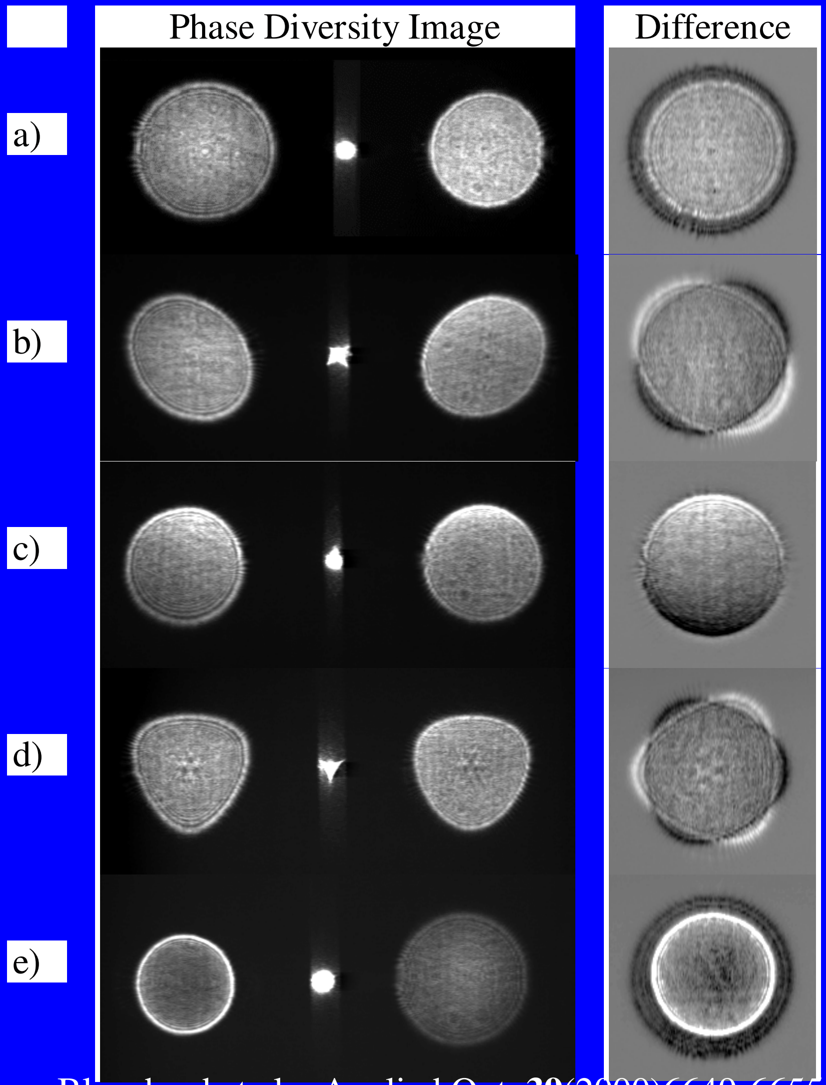
a) defocus

b) astigmatism

c) coma

d) trefoil

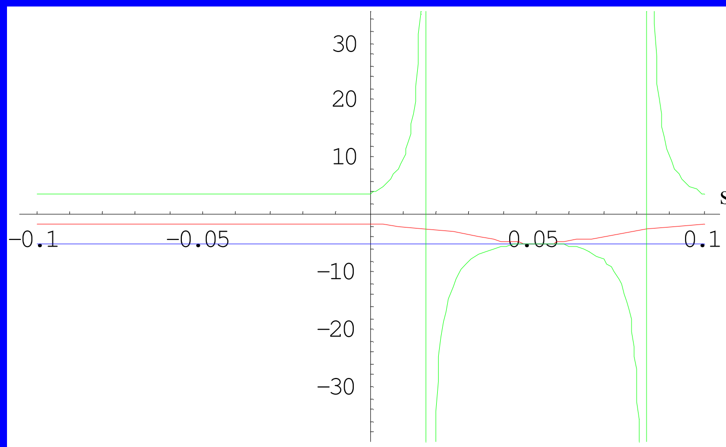
e) spherical aberration



Blanchard et al Applied Opt. **39**(2000)6649-6655

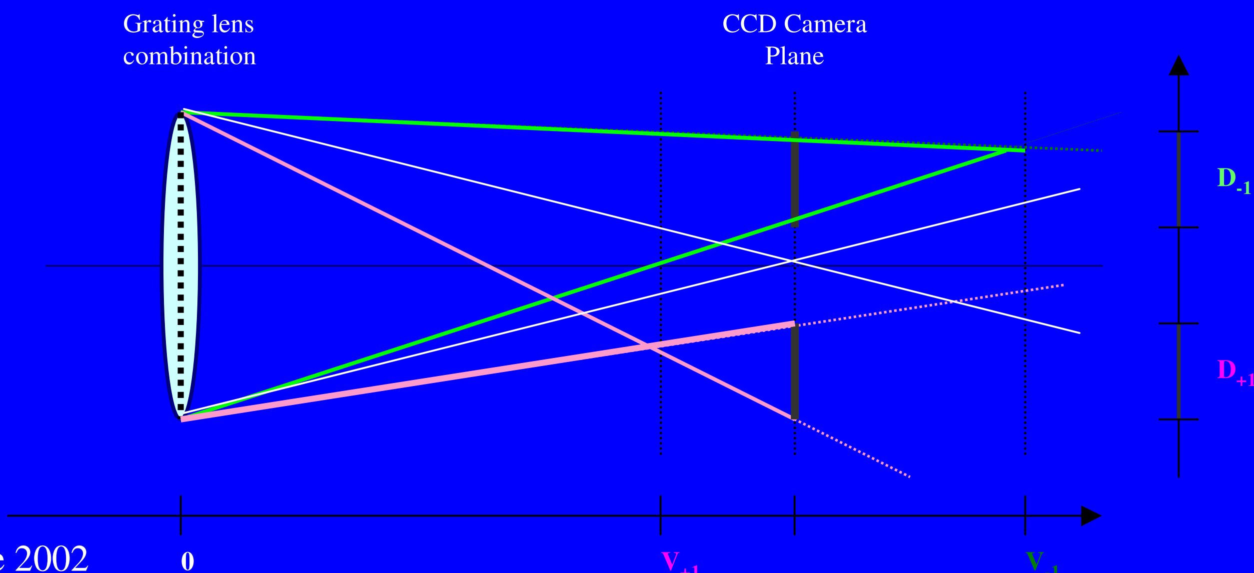
19June 2002

How critical are placements?

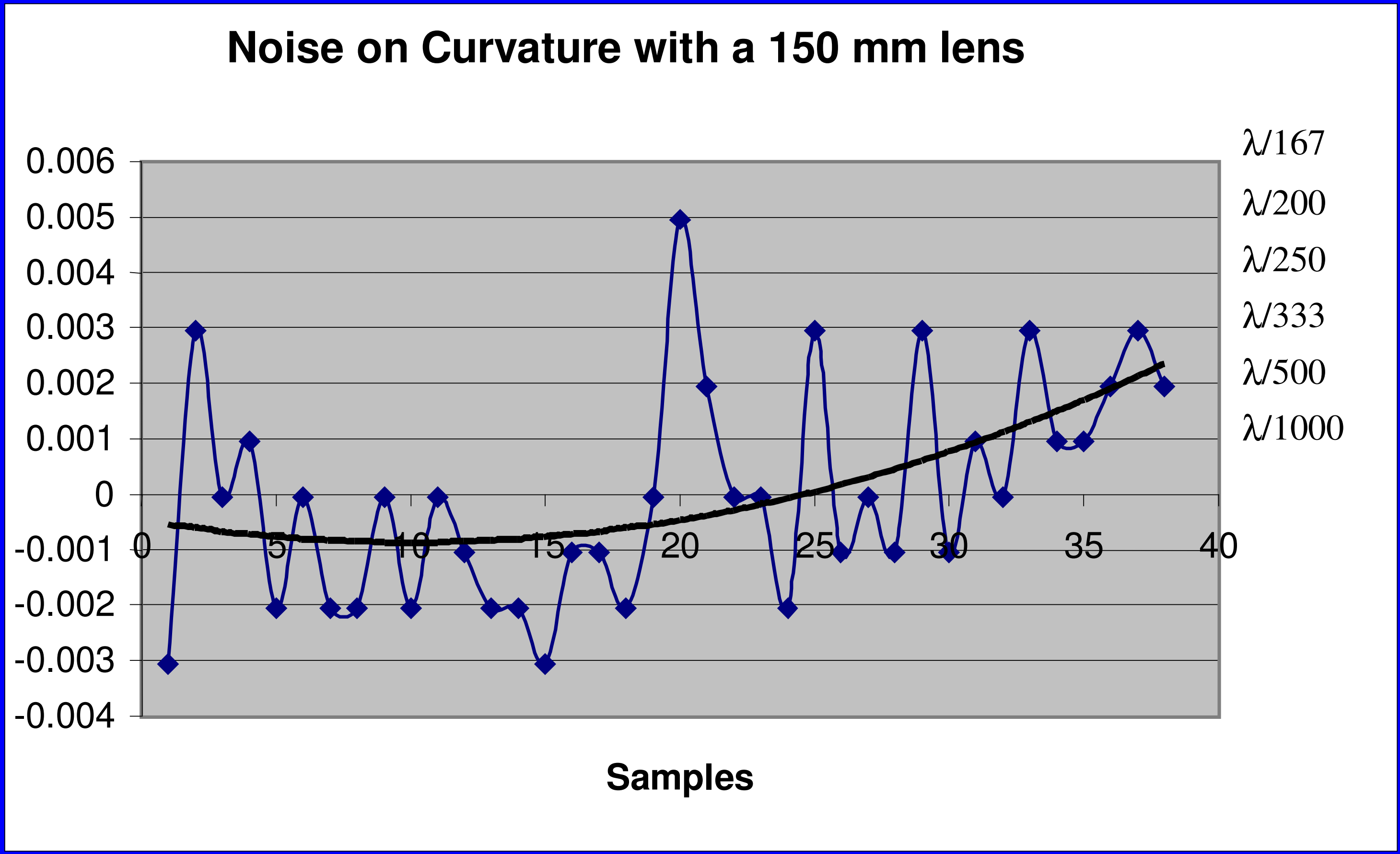


$U_0 = -5$ m [redacted] is U_{+1} and Green is U_{-1} [redacted]

Provided that source is not in the near field and the IMP® is in front of the lens, the positions of the planes are reliably defined



Best Results



What of remaining effects?

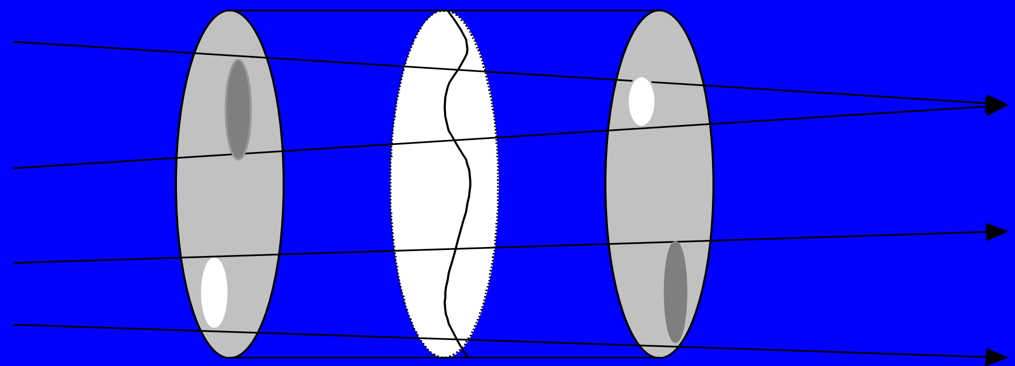
- Background subtraction
  - ➢ Vital for accuracy
  - – Darkened lab but computer screens in lab vary in brightness
  - – No spectral filters used (avoid unknown aberrations)
- No attempt in lab to control temperature
- No attempt in lab to control air currents
- Jitter on translation stage
- Numerical output to only 3 decimal places

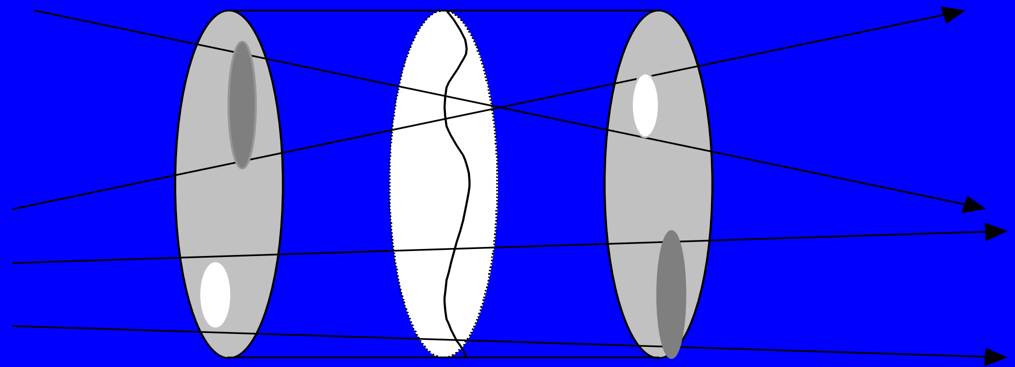
Restrictions

- Intensity assumed uniform
- Wavefronts continuous with continuous 1st derivative
- Close measurement plane to approx derivative

- Metrology implies laser illumination, implies speckle
- Disadvantage if measuring man-made surfaces
- Dynamic range is restricted

Wavefront Sensing Schematic





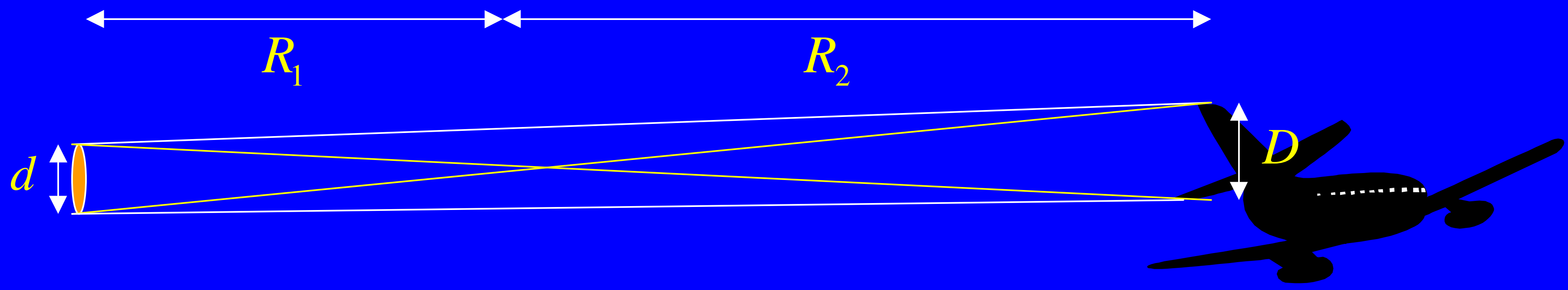
Other diversity Kernels?

- Why restrictions?
- But ITE is an approximation
- What is special about phase diversity/ wavefront curvature?

- ➢ Solution constraints on ITE
- ➢ iterative solutions reconstruct discontinuities
  - ➢ Easy interpretation
  - ➢ Local measurement
  - ➢ Fast implementation

Greater flexibility is likely if the approximation of the ITE is avoided and other kernels investigated for WFS - e.g. RSC

Anisoplanatism



- By similar triangles $$\frac{d}{R_1} = \frac{D}{R_2} \Rightarrow \frac{R_2}{R_1} = \frac{D}{d}$$

terrestrial imaging is almost always severely anisoplanatic

Conclusions

- Phase-diversity WFS based on IMP® technology is capable of $\lambda/1000$ accuracy
- Accuracy is λ independent, best 0.7nm
- Control of background subtraction is greatest problem with present arrangement
- Greater flexibility likely using more complete description and other diversity kernels