Pupil Replication

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PREDRIS

- Pupil Replication for Extreme Dynamic Range Imaging Spectroscopy
- Start: 1 April 2005
- Two years, funded by PPARC
- Publication: Pupil Replication for Exo-Planet Imaging; ApJL, 10 January 2005 (referenced in Nature).
- This presentation: includes new unpublished results.

Overview

- Introduction to Pupil Replication
- Characterization and analysis
- Error Assessment
- Experimental proof of principle
- Simulation of a Coronagraph
- Comparison with Hyper Telescope principle

Context

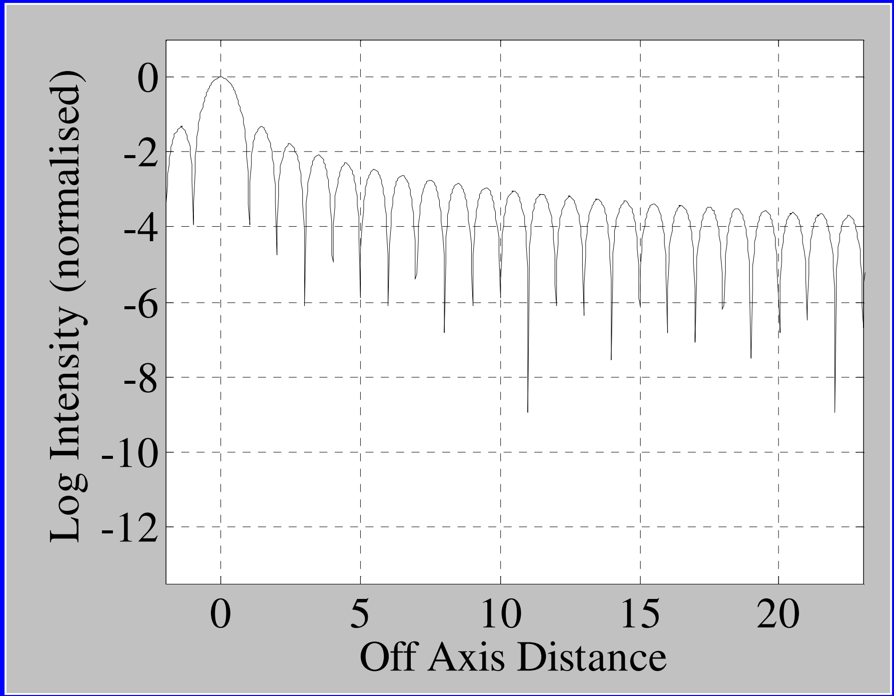
- Exoplanet indirect detection since 1995
- ESA Cosmic Vision 2020 (April 2005):
  - fundamental physics
  - cosmology
  - solar system
  - planetary formation, life, exoplanets
  - => direct detection of exoplanets
  - => proposals due early 2007

Numbers

- Dynamic range starflux / planetflux:
  - 10^{10} (visual, reflected), 10^{6}(infrared, thermal)
- A planet at 1 AU around a star at 20 pc seen with 3 m telescope
  - at 36 λ/d at 600 nm (stardisk => 0.2 λ/d)
  - at 2 λ/d at 10 micron
  - flux: one hour,V-band, efficiency 50% =>
    - ~500 photons (planet around 0 magnitude star)
    - ~.05 photons (planet around 10 magnitude star)

Star flux problem

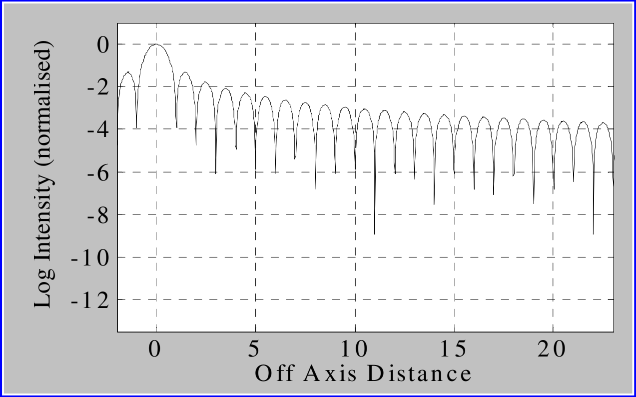
- Flux from host star dominates exoplanet:
- Techniques to suppress scattered host flux:
  - coronography
  - pupil apodisation
  - interferometry



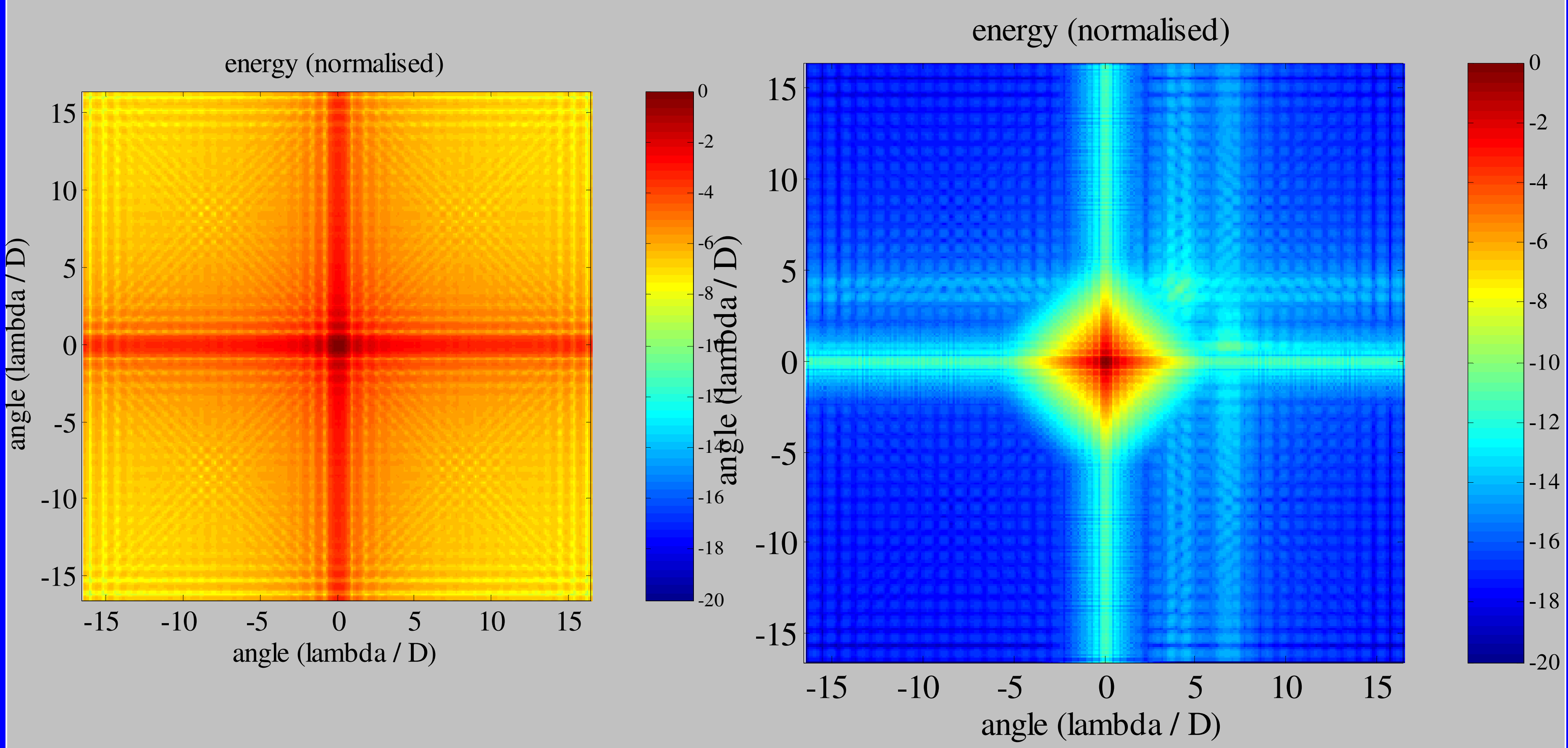
diffraction limited PSF, x-axis in λ/d

Inner Working Angle

- Inner working angle (IWA): the minimum angular distance from the star at which a planet can be detected.
- All techniques need image-plane masks
  - mask size is typically several λ/D
  - side-lobe suppression often increases star image
  - increasing IWA makes earth-like detection harder
- => decrease size of star image
  => pupil replication

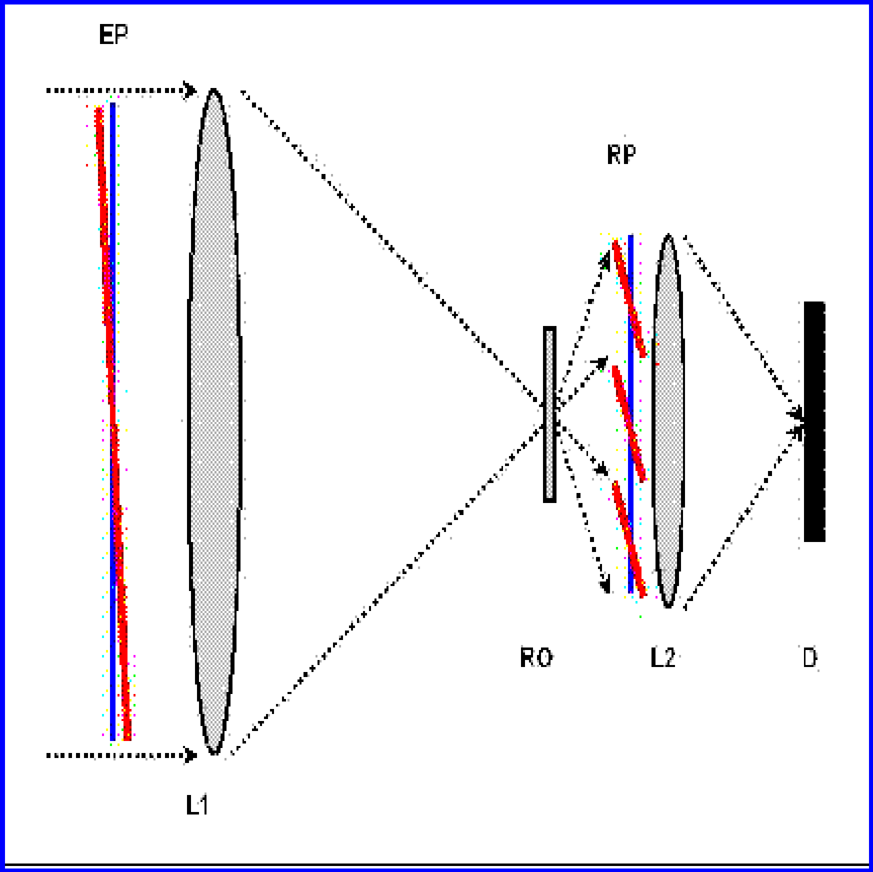


Pupil Replication with Apodisation



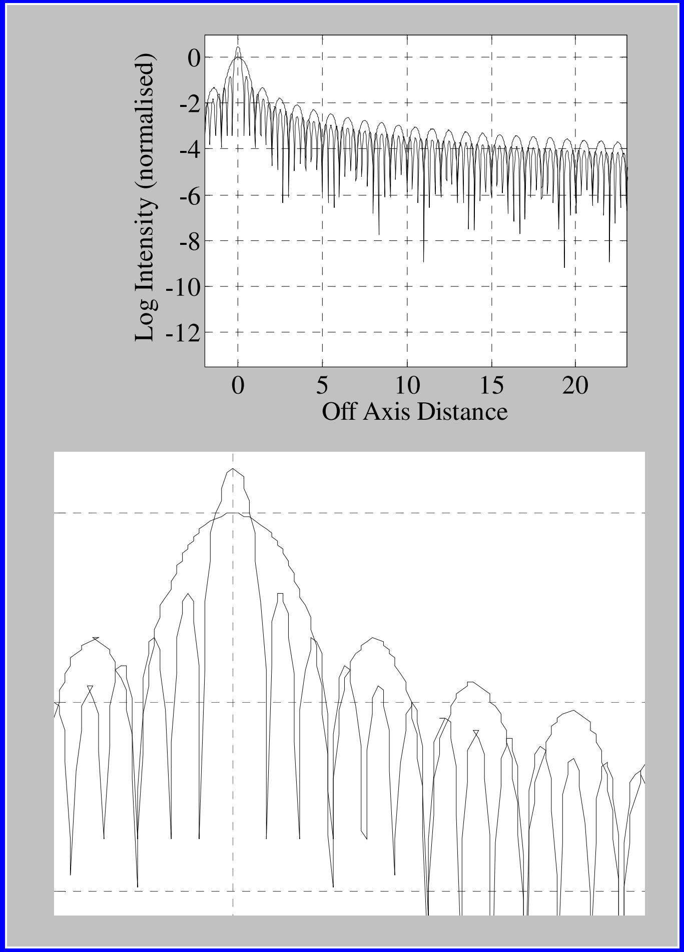
Pupil Replication

- Axial wavefront is continuous (blue), like larger telescope.
- Non-axial wavefront is discontinuous (red), and appears like a blazed grating
- Etendue is preserved (pupil area * solid view angle)



Expected Results

- Consider 3-fold replication in 1-d
- Axial pupil wavefront is 3x wider and 3x fainter
- Image of unresolved axial star is 3x narrower and 3x brighter
- Sidelobes are 3x fainter
- Image stop to remove star flux can be smaller



Expected Limits

- Violation of homothetic mapping
  - position and orientation of the replicated images is not an exact scaled copy of the unreplicated image
- => Non-isoplanatic imaging
  - PSF dependent on the viewing direction.
- => Requirement:
  - Maximum angular diameter of star + telescope pointing error < some limit: θ

Example

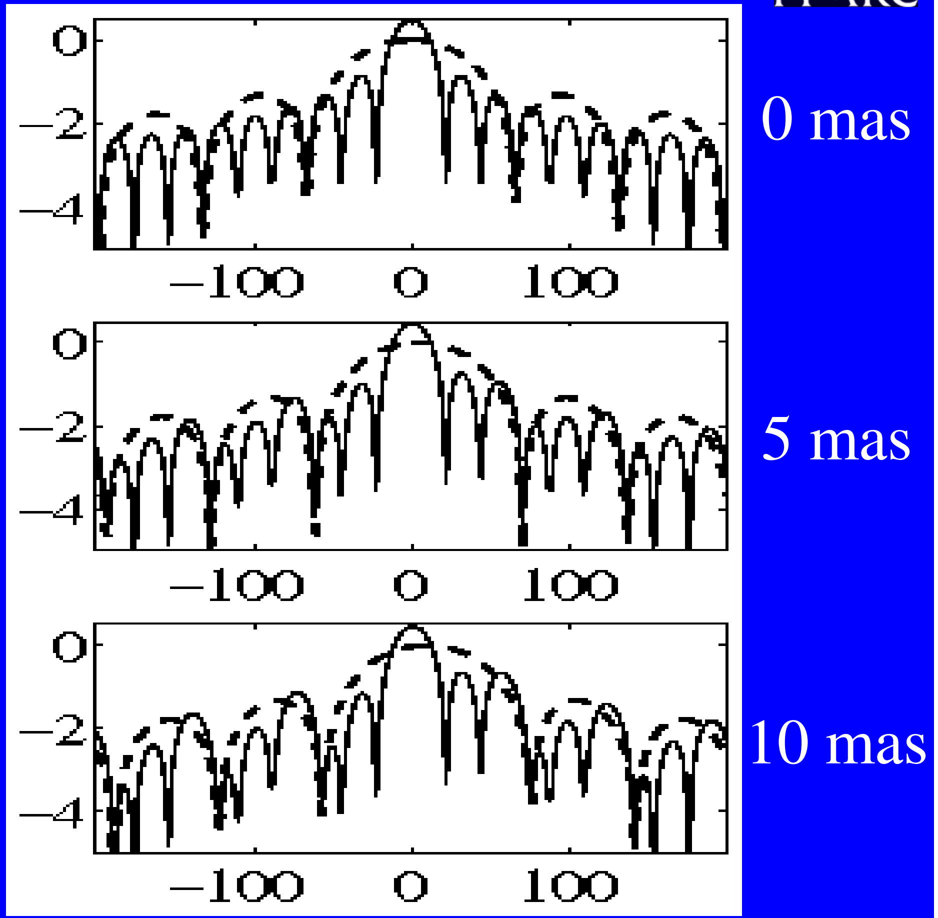
- General criterion: max wavefront error < $\lambda/4$.
  - => For N-fold replication of aperture diameter D and at wavelength λ:

$$\theta < \frac{\lambda}{4D(N-1)}$$

- Example:
  - 2.5m diameter pupil
  - 800nm wavelength
  - 3-fold pupil replication
  - $\theta < 5$ mas
- At angles smaller than θ the image quality should be ‘good’ (includes pointing error).

Simulation

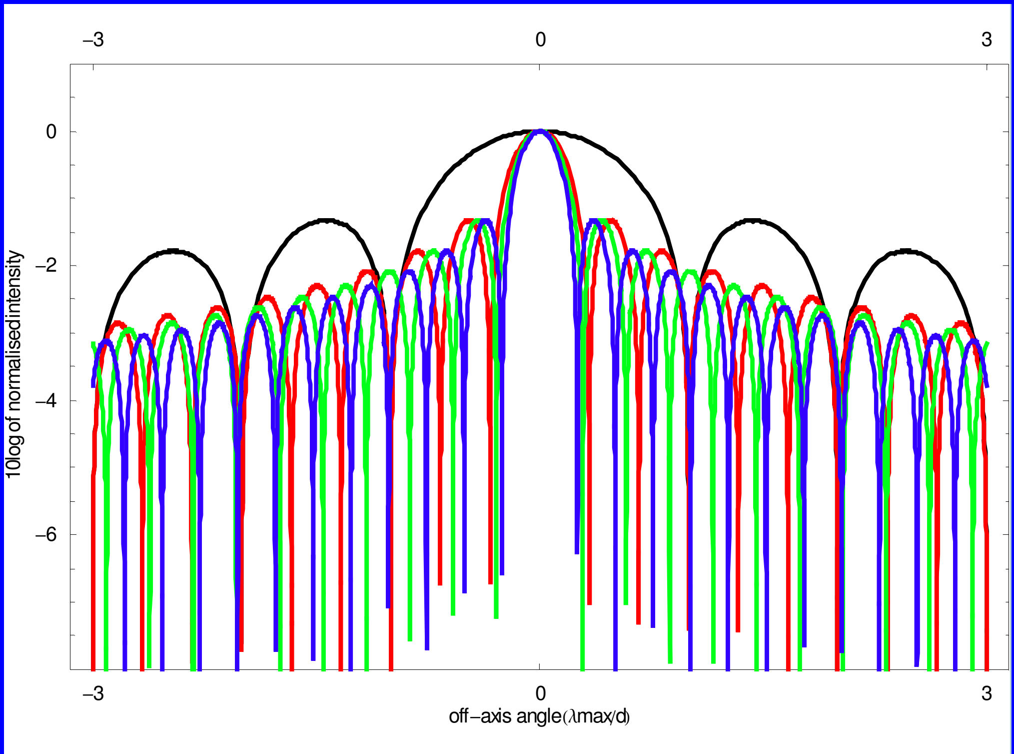
- Broken line is un-replicated image (diffraction limited)
- Solid line is replicated image
- Replicated image is modulated by un-replicated image, as expected



y: log(int) x: mas

Broadband

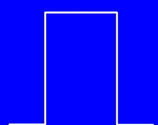
- Single pupil, 1000 nm (black)
- 3x Replicated pupil
  - 1000 nm (red)
  - 872 nm (green)
  - 760 nm (blue)



Analysis (1)

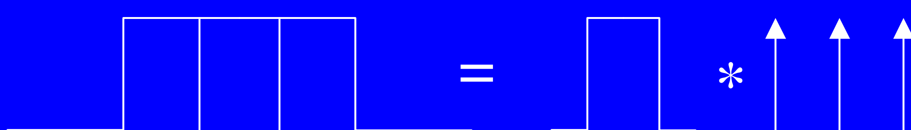
in 1 dimension, plane wave on axis, hard edged pupil:

wave + pupil: | imaged:



FT

$\sin(x)/x = \mathrm{sinc}(x)$



FT

$\mathrm{sinc}(x)\ (1+2\cos(x))$

Analysis (2)

- Input wave at angle α, d = pupil diameter
  A = amplitude constant

$$wave(x) = Ae^{2\pi(x-\sin(\alpha))/\lambda}$$

- Amplitude of imaged plane wave = T

$$T = A\mathrm{sinc}\left(2d\pi\left(x-\sin\left(\alpha\right)\right)/\lambda\right)$$

- PSF = $|T|^2$

Analysis (3)

- Three replications:
  - single pupil convolved with 3 delta functions
  - Fourier(cosine) = 2 delta functions
  - => sinc multiplied in Fourier domain (image) with 1 + 2cos(x)

$$T = A\left(1+2\cos\left(2d\pi x/\lambda\right)\right)\mathrm{sinc}\left(d\pi\left(x-\sin\left(\alpha\right)\right)/\lambda\right)$$

  - Note: Cos( ) is not dependent on angle

Two dimensions

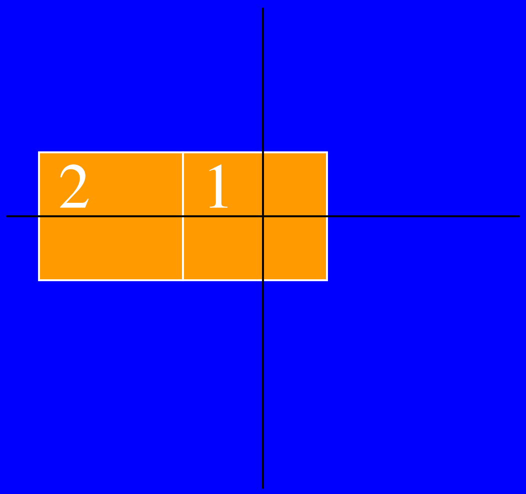
- Square pupil (seamless replication) then: 2-D amplitude T

$$T = T_x T_y$$

- T for two replications 1 and 2
  - 1 is on axis, 2 is adjacent

$$T = T_{x1} T_{y1} + T_{x2} T_{y2}$$

  - errors imposed on 2



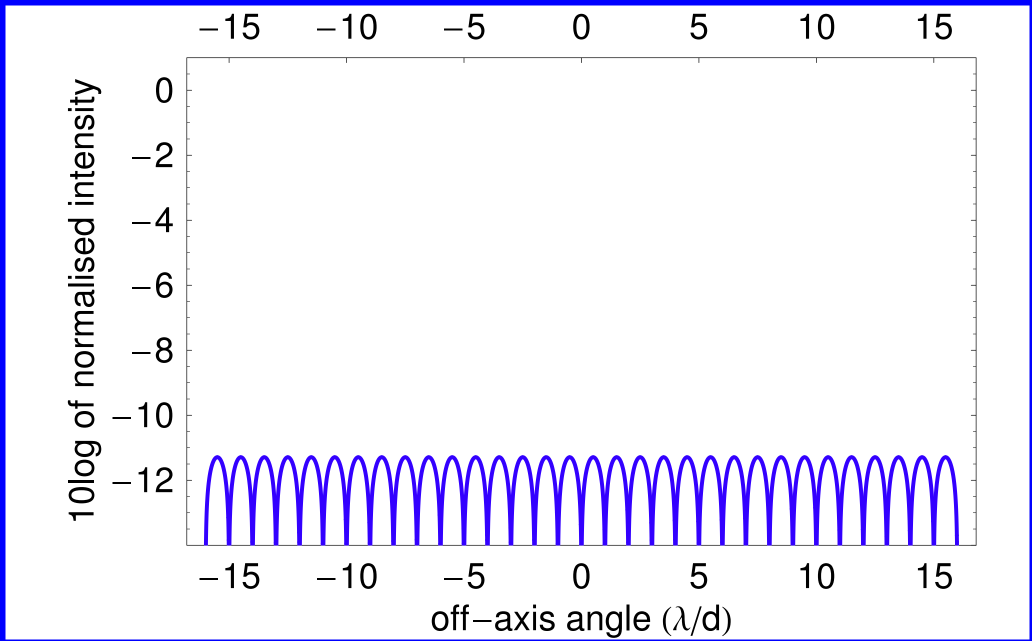
Errors

- Pupil 1: T_{1x} and T_{1y} are sinc functions
- Pupil 2 has errors in x (and y) direction:
  - shift s (and shear h):
  - piston p
  - tilt f (and tip g)
  - T_{2x} (and similar for T_{2y}):

$$Ae^{i2\pi\left(p+\left(x/\lambda\right)(d+s)\right)}\operatorname{sinc}\left(d\pi\left(\left(x-\sin(\alpha)\right)/\lambda-f\right)\right)$$

Evaluation

- 2-D analyses; horizontal crosssection
- If PSF(no error) - PSF(with error) < 10^{-10}
  - example: shift = d 10^{-5} => 1 micron for 1 cm pupil
  - difference image =>



Examples

- Similar results below 10^{-10} for
  - shear = 10^{-5} d; piston = 10^{-6} λ; tip = 10^{-5} λ/d; tilt = 10^{-5} λ/d; jitter = 10^{-6} λ/d
- No apodisation or star stop applied.
- Is this peculiar to PR ? =>
  - Error sensitivity assessed this way is for single pupil of the same order of magnitude.

Software Simulations

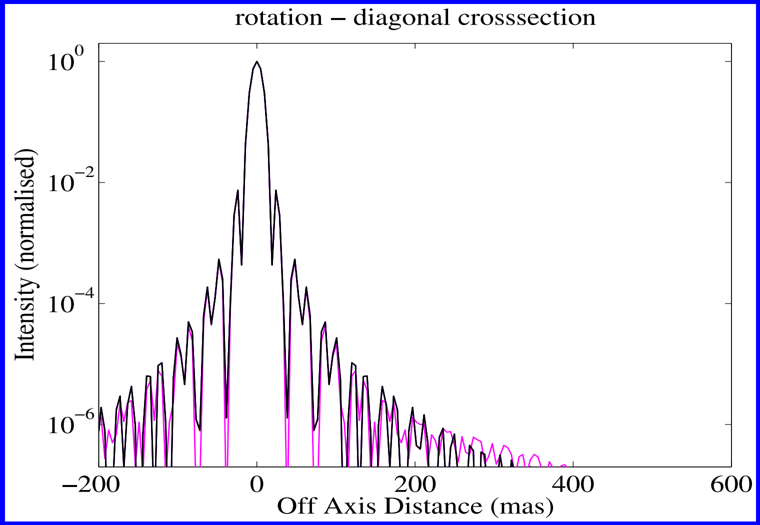
- 2-D; horizontal crosssections shown
- Simulations of errors due to
  - shift, shear, piston, tip, tilt, jitter
  - agree with analysis within 10^{-14}
- Further simulations:
  - Rotation error, (de-)magnification error, random amplitude errors, random phase errors.
  - Extended sources

Simulation: Procedure

- Setup: two square pupils like in analysis
  - wave - pupil - errors - FT - grating - FT^{-1}- pupil - apodisation - FT - star stop - sampling correction - modulus^2 - store
    => this for each point source and wavelength and sum the results
  - Adapt for each wavelength: size of pupils, of apodisation, of star stop, of grating.
  - NB: maximum # pixels limited; sampling correction important; accuracy $\sim 10^{-14}$

Example

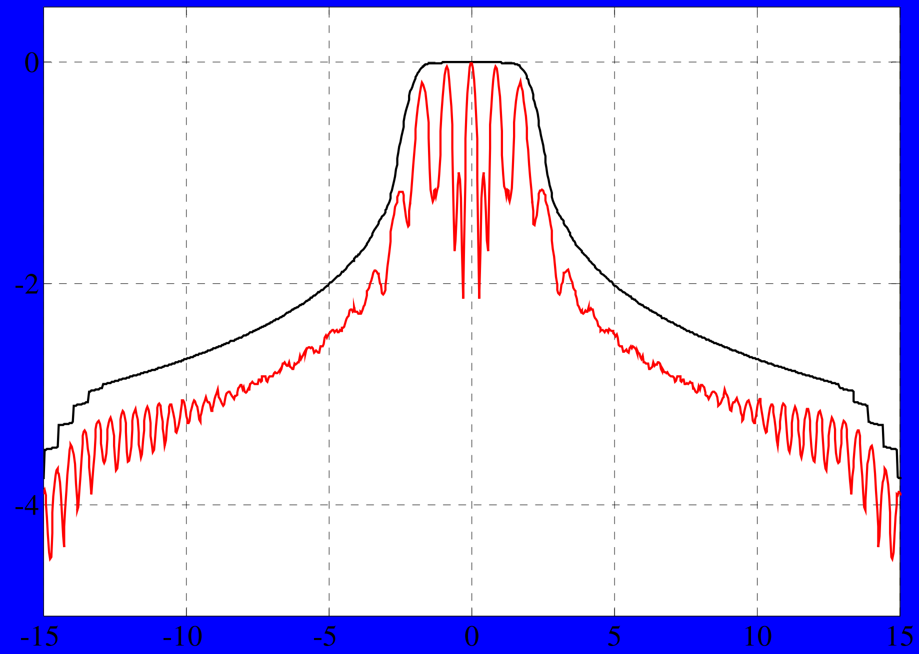
- Rotation error, max = 21 degrees (magenta), and no error (black), diagonal crosssection.



- Similar results for (de-)magnification error.
- Random errors addressed later.

Extended source

- Simulation using 500 point sources in broadband (8 wavelengths), source = 5λ/d
- The cosine term marks the profile (?)
  - – black: single pupil
  - – red: 3 replications
  - – equal normalisation
- Important when star > λ/2d
  (sun at 20 pc with 9 m telescope at 600 nm)



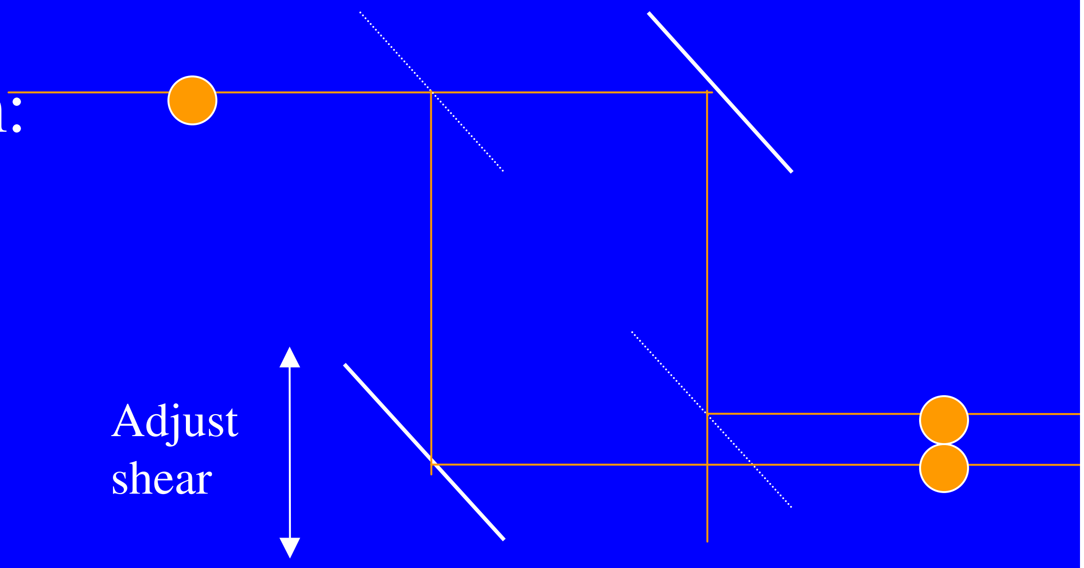
Half-time

- So far:
  - Pupil Replication as such
  - Analysis, error behaviour

- To come:
  - Experiment
  - Coronagraph
  - Hyper telescope?

Experiment

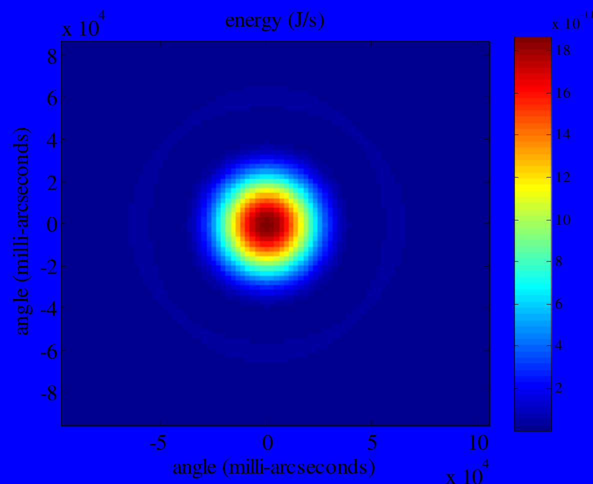
- A first experiment to verify principle
- Interferometer beam:
  - 2 coherent beams
  - Monochromatic
  - Circular pupils with Gaussian beams
- Effect in one dimension only

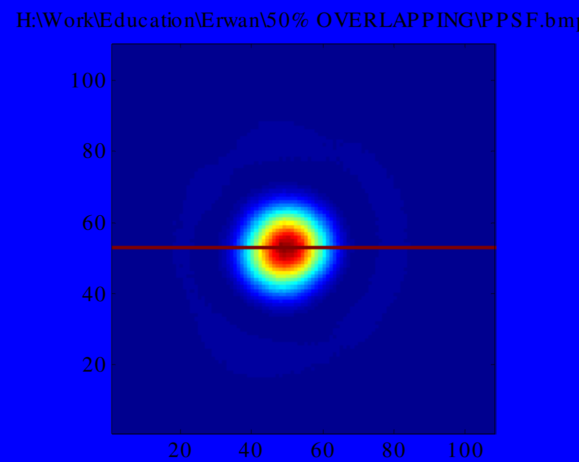


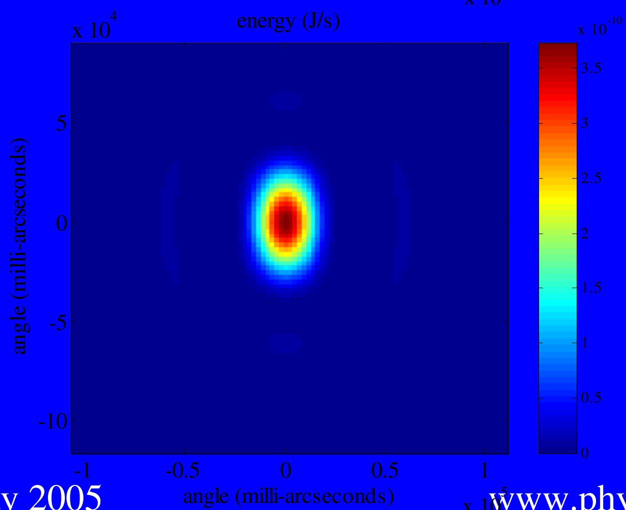
PSF, pupils: 50% overlap

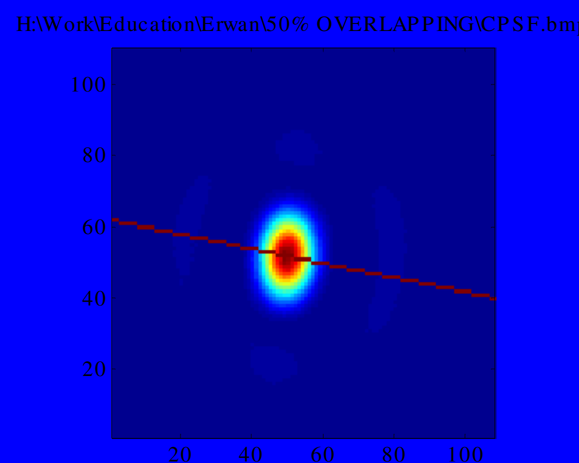
simulation

experiment







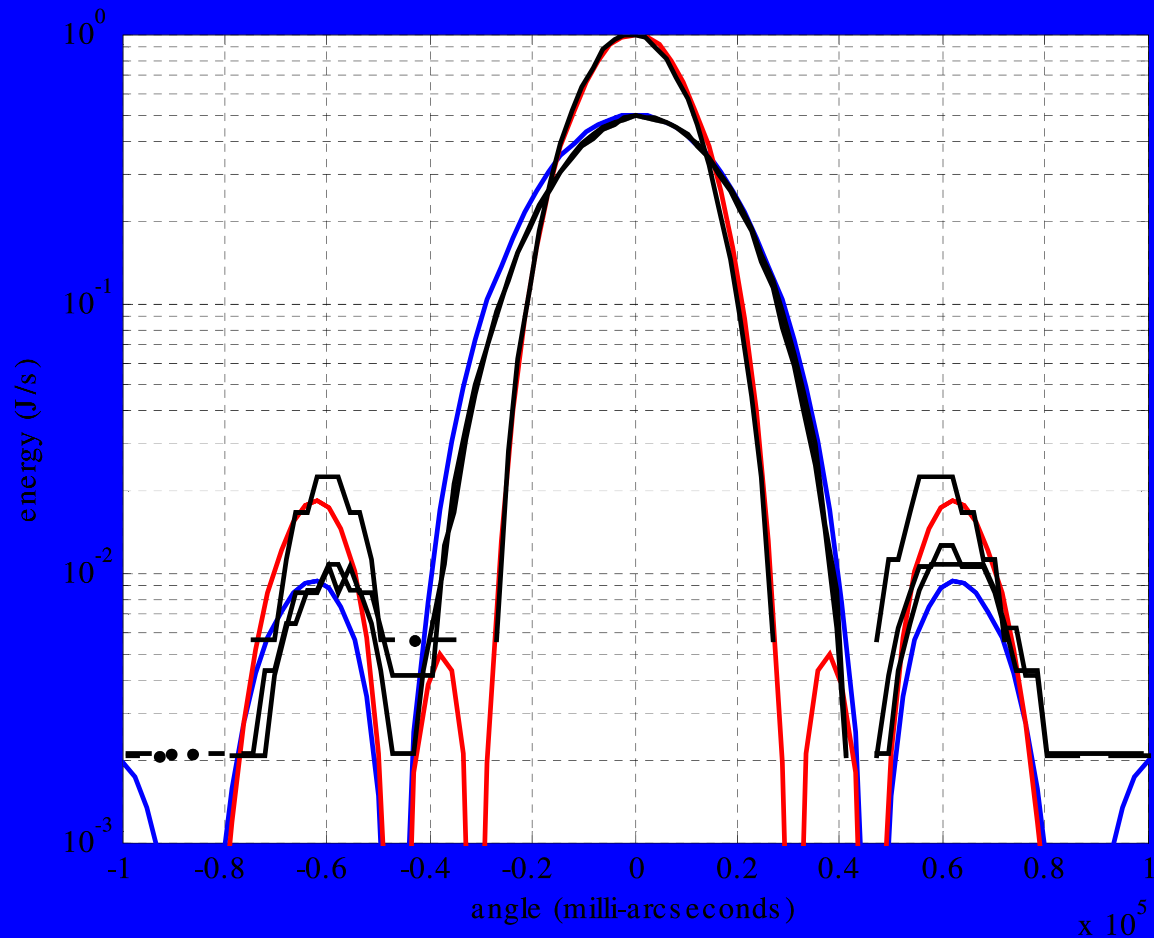


Cross-sections

Blue: simulation, unreplicated

Red: simulation, replicated

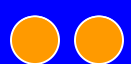
Black: experimental

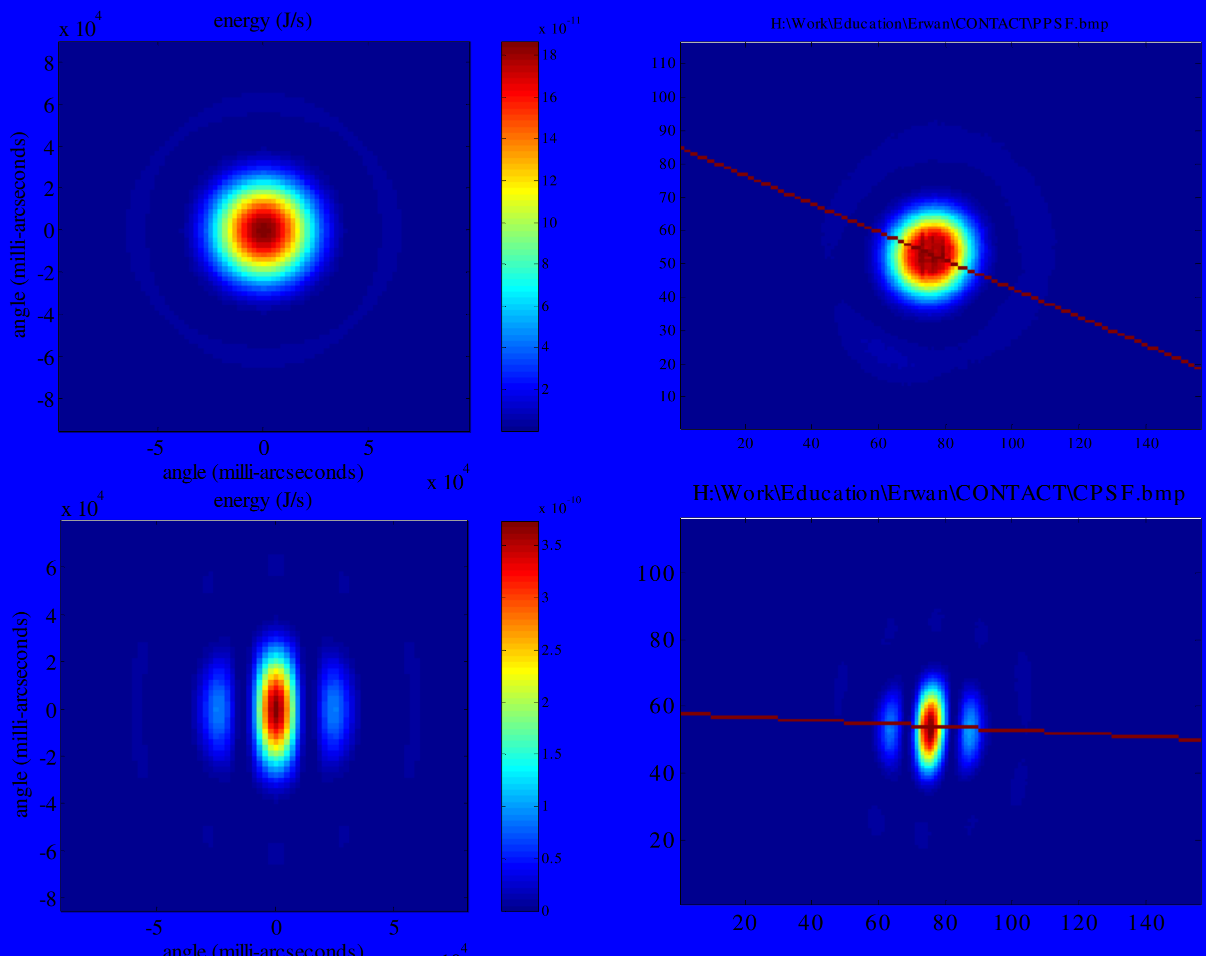


PSFs, pupils: no overlap

simulation

experiment





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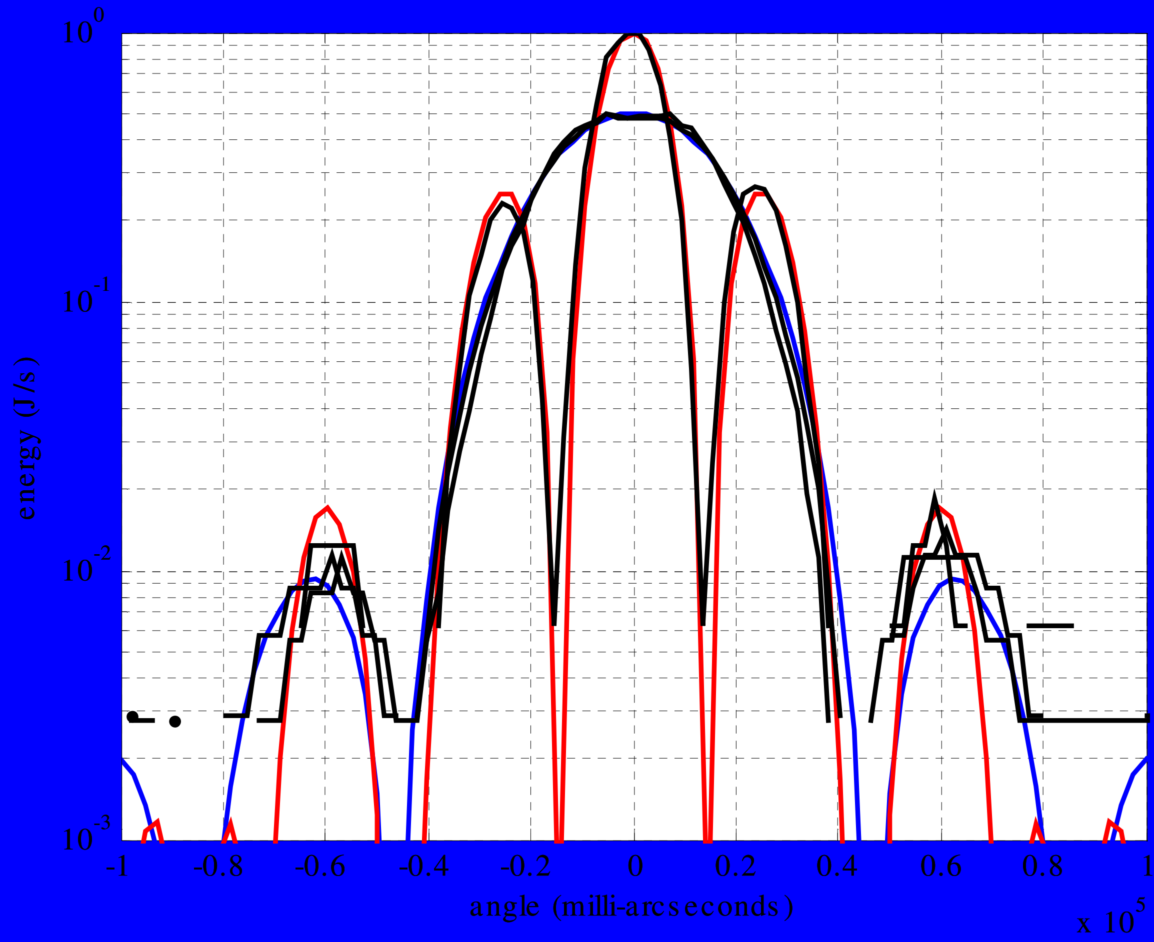
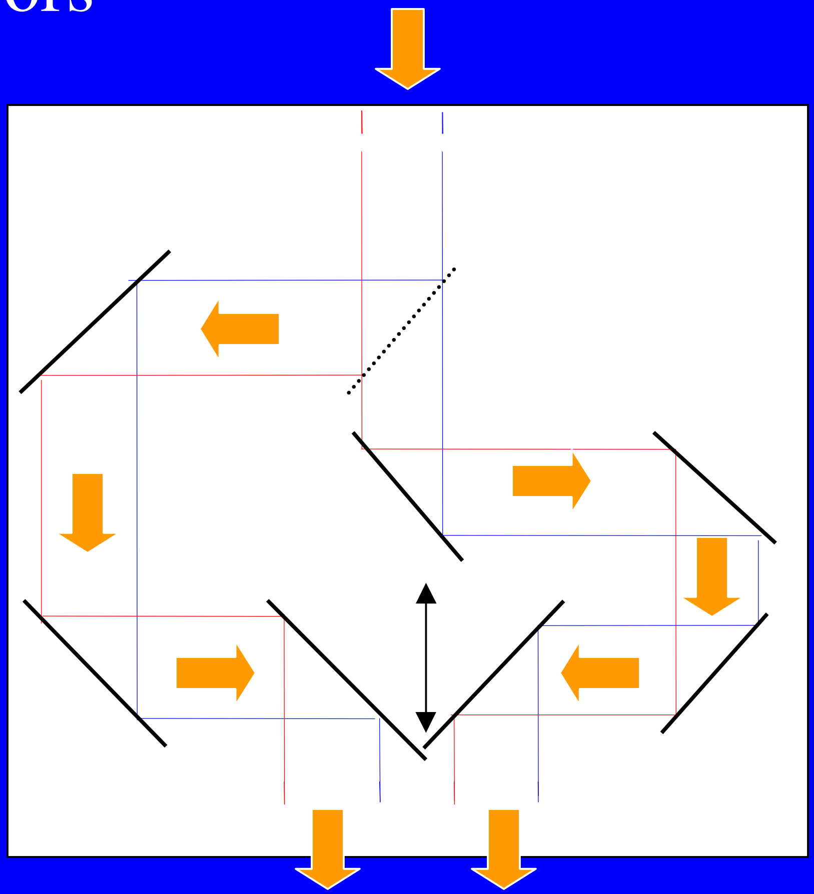


Replication

- Use 1 beamsplitter and mirrors
- Equal:
  - # reflections in each arm
  - polarisation
  - optical path length
- Adjust last prism to vary replica separation
  => no shift error
- Can be cascaded
- Can be monolithic



Pupil Apodisation

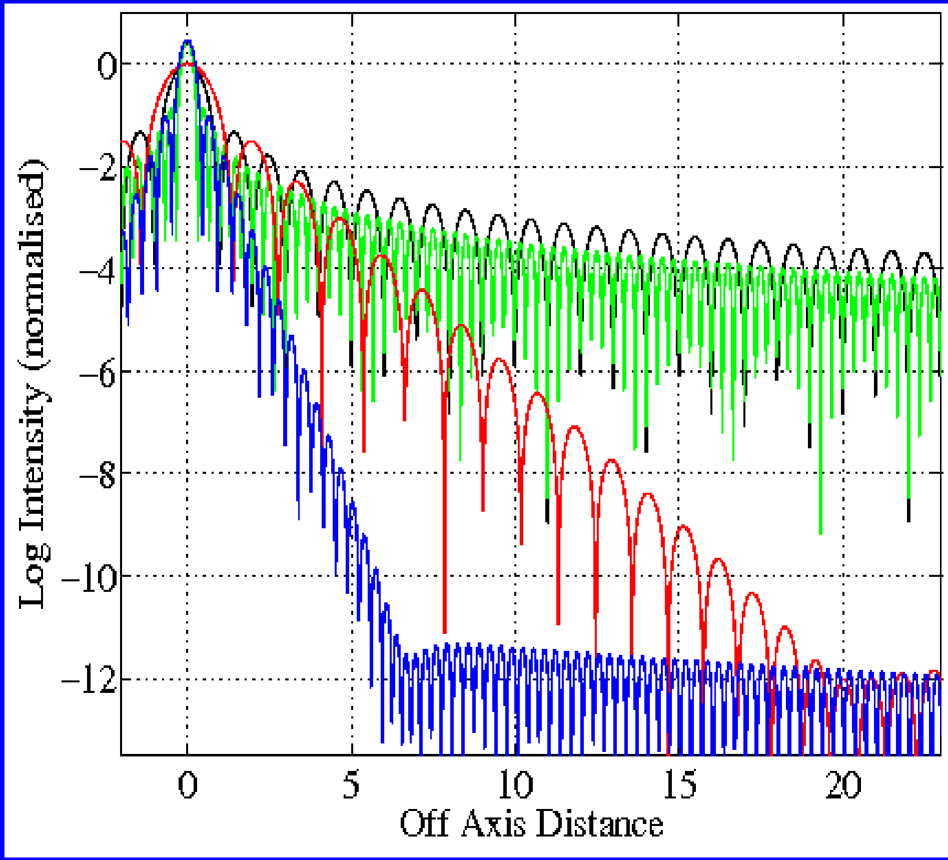
- Used simple not optimised function for attenuation B:
  - Super Gaussian
  - x = off-axis angle
  - c is adjusted to attenuate by 10^{-4} at pupil edge (both un- and replicated)

$$B(x) = e^{-\left(x/c\right)^{8}}$$

- Expect to
  - broaden the image core
  - reduce side-lobes

Simulation

- Unapodised
  - Black: unreplicated
  - Green: 3-fold replication
- Apodised
  - Red: unreplicated
  - Blue: 3-fold replication
- Efficiency:
  - 65% throughput
- Sidelobes suppressed



Units: λ/D

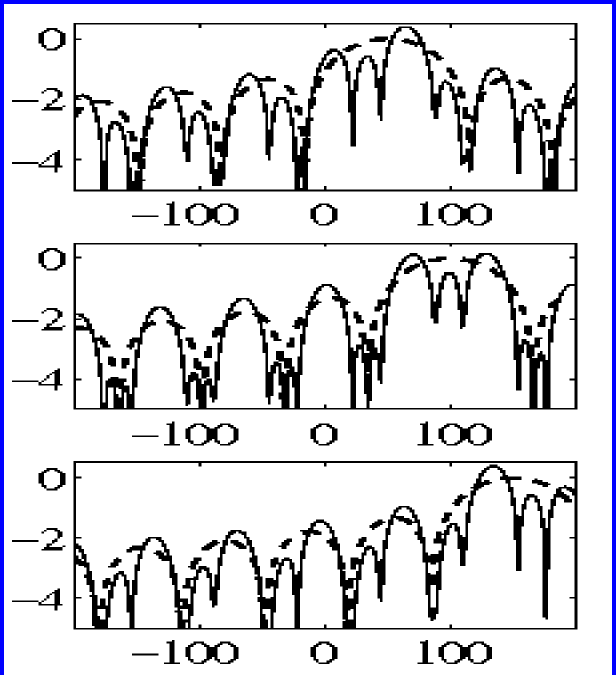
Off-axis effects

- Off-axis (planet) images will be distorted:

50 mas

100 mas

150 mas

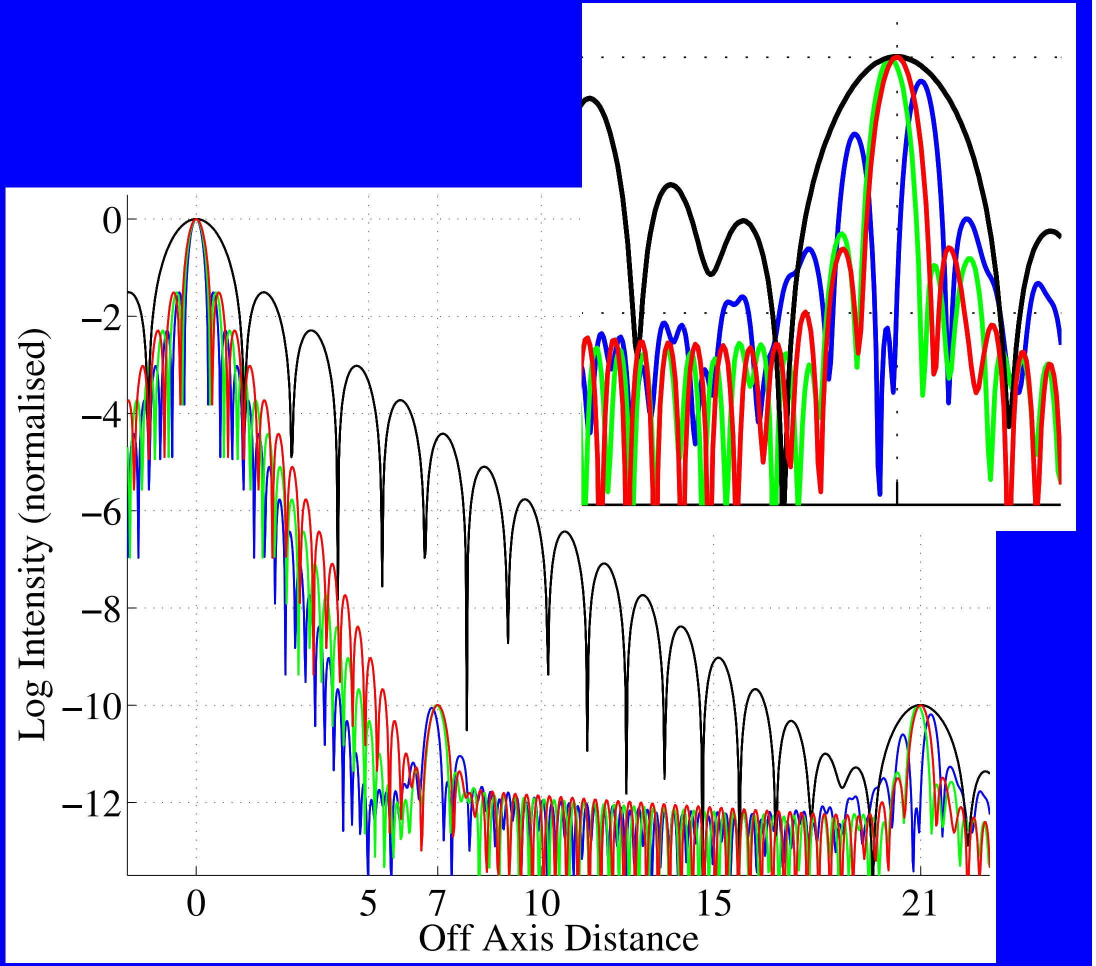


y: log(int) x: mas

- Wavelength dependent => chromatic effects.
  - ➢ will these obstruct spectroscopy?
  - ➢ what other effects will this have - SNR?

Simulation

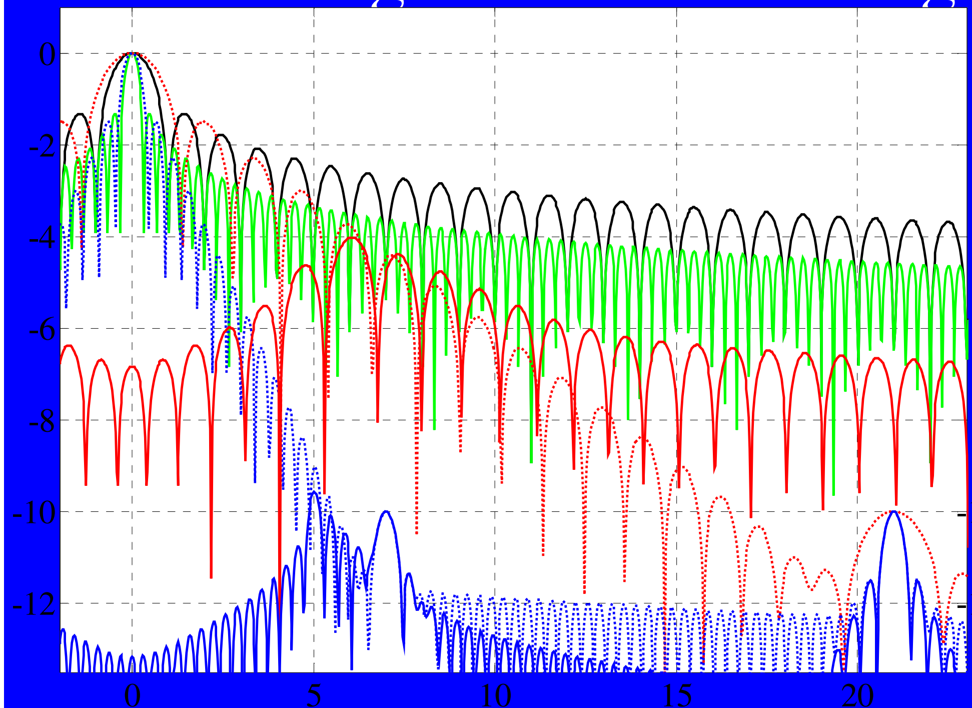
- Planets at 7λ/d and 21λ/d, each 10^{-10} brightness of host star
- 3 colours shown 760nm (blue), 872nm(green), 1000nm (red&black)

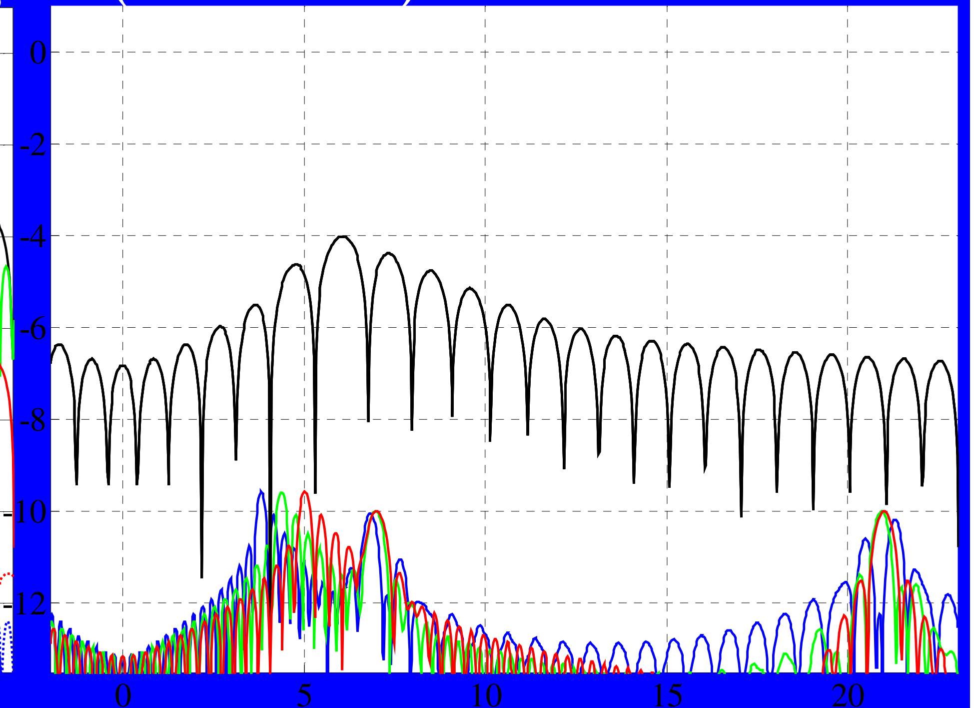


Units: λ/d (at 1000 nm)

Star Stop added

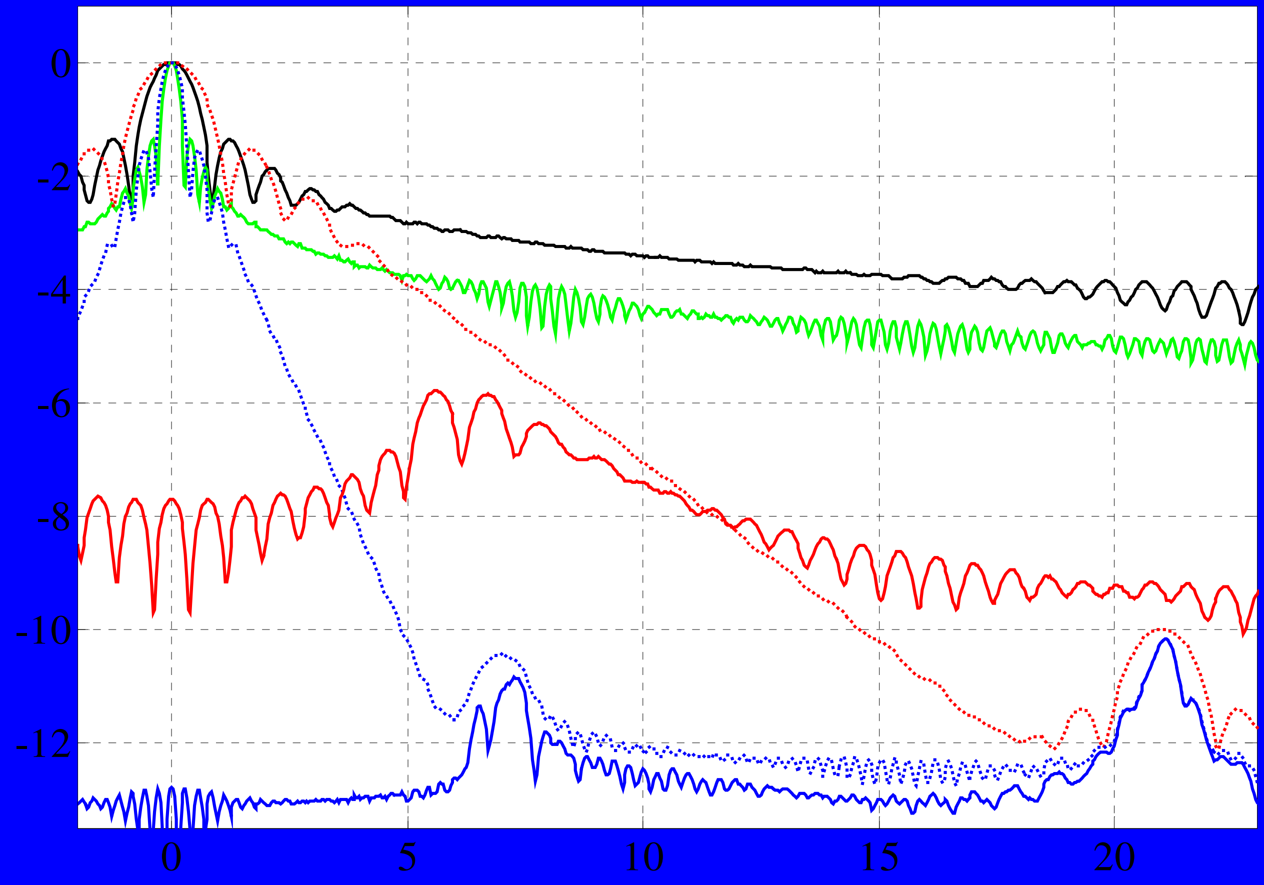
– 5λ/d half-width solid star stop in image plane
– second pupil added, no Lyot stop
– Unreplicated: solid red, replicated: solid blue
– right: three wavelengths (coloured)





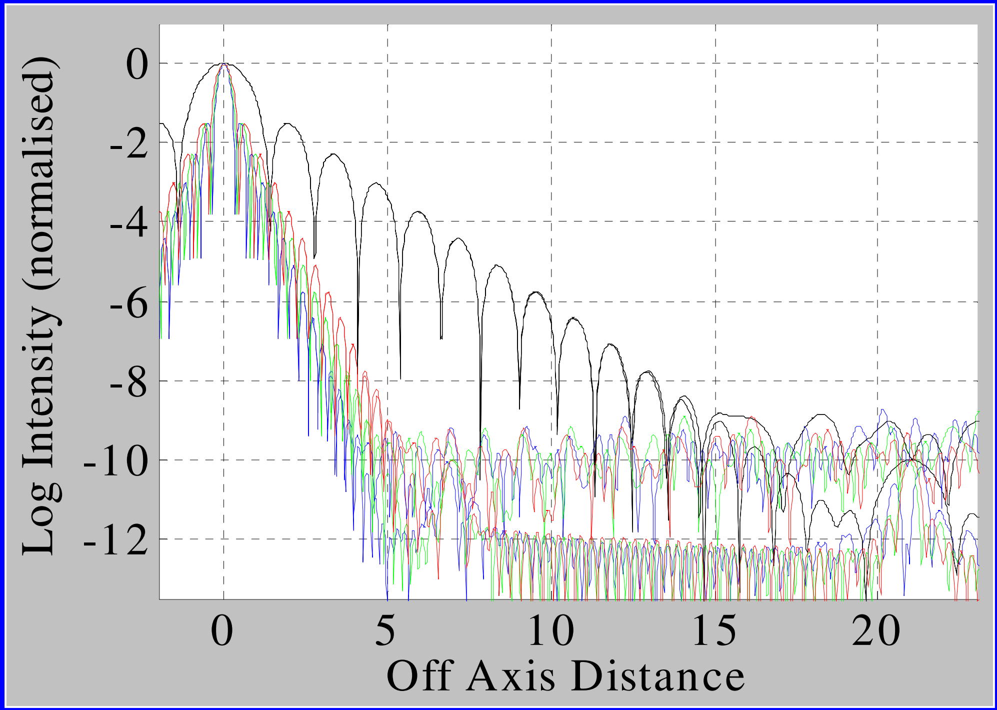
Coronagraph

- Broadband (V-band in 8 wavelengths)



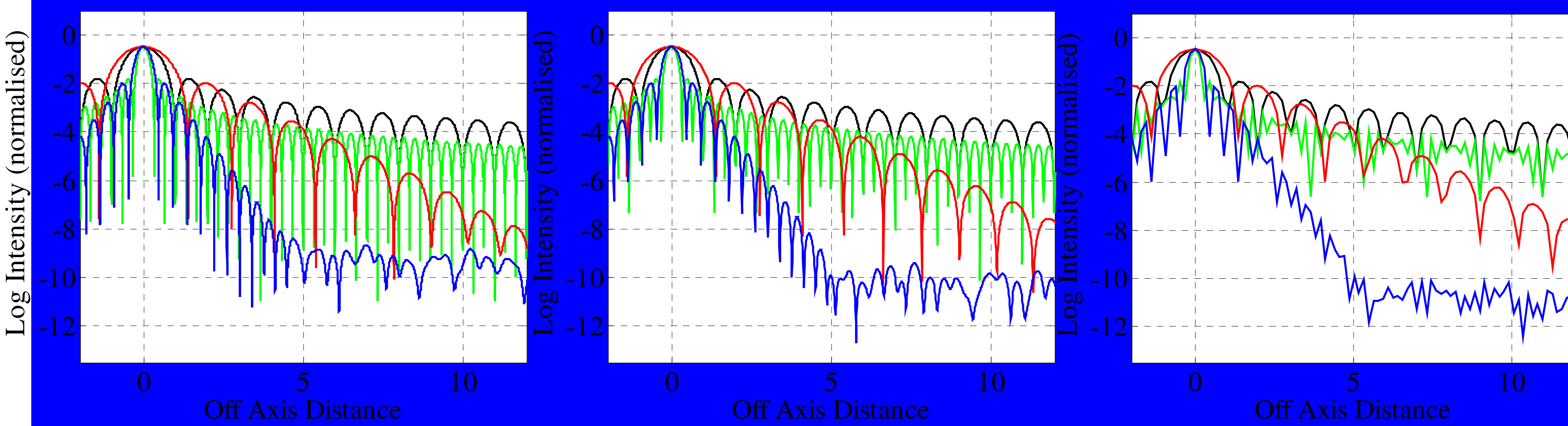
Sensitivity: amplitude

- At this accuracy, amplitude errors matter
- Simulations suggest that better than 0.1% is required to image exo-earths
- Simulation:
  - random amplitude error 10^{-3} max.
  - pixel size ~ 1 cm



Pixel size

- 10-3 random amplitude errors, with 25, 251, 2501 pupil pixels (100, 10, 1 mm/pixel)

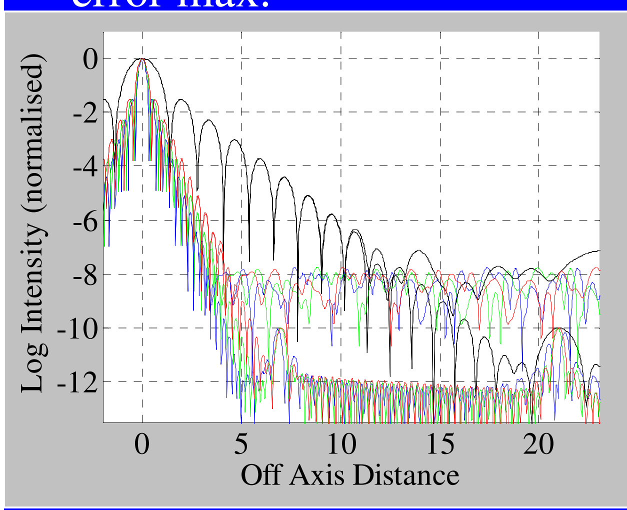


Sensitivity: phase

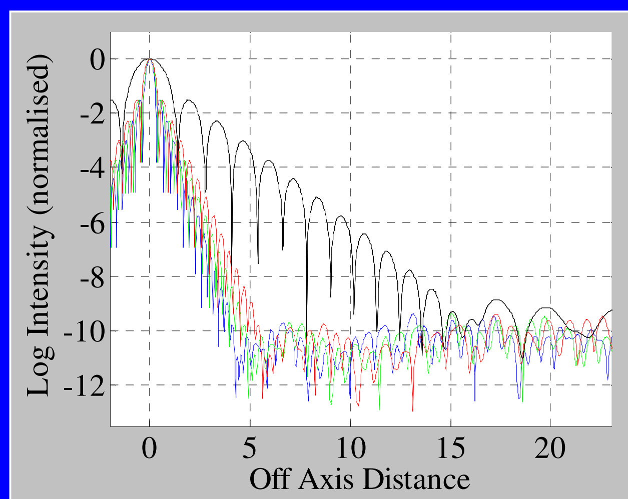
- Higher sensitivity to phase errors
- Simulation suggests that better than λ/10000 is needed to image exo-earths
- Pixel size 1 cm

Random phase error max.

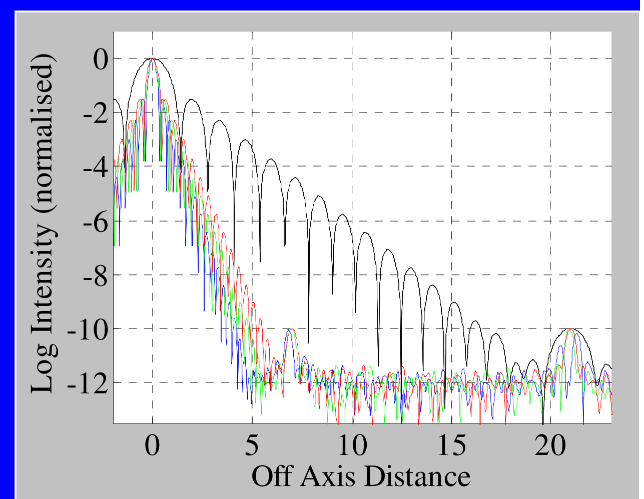
$10^{-3}\lambda$



$10^{-4}\lambda$



$10^{-5}\lambda$



Hyper Telescope?

- Pupil Replication =/≅/≠ Hyper Telescope?
  - Assessment of the hyper telescope principle
  - using analysis like before
  - simplified situation
  - characterisation only
  - adding to discussion...

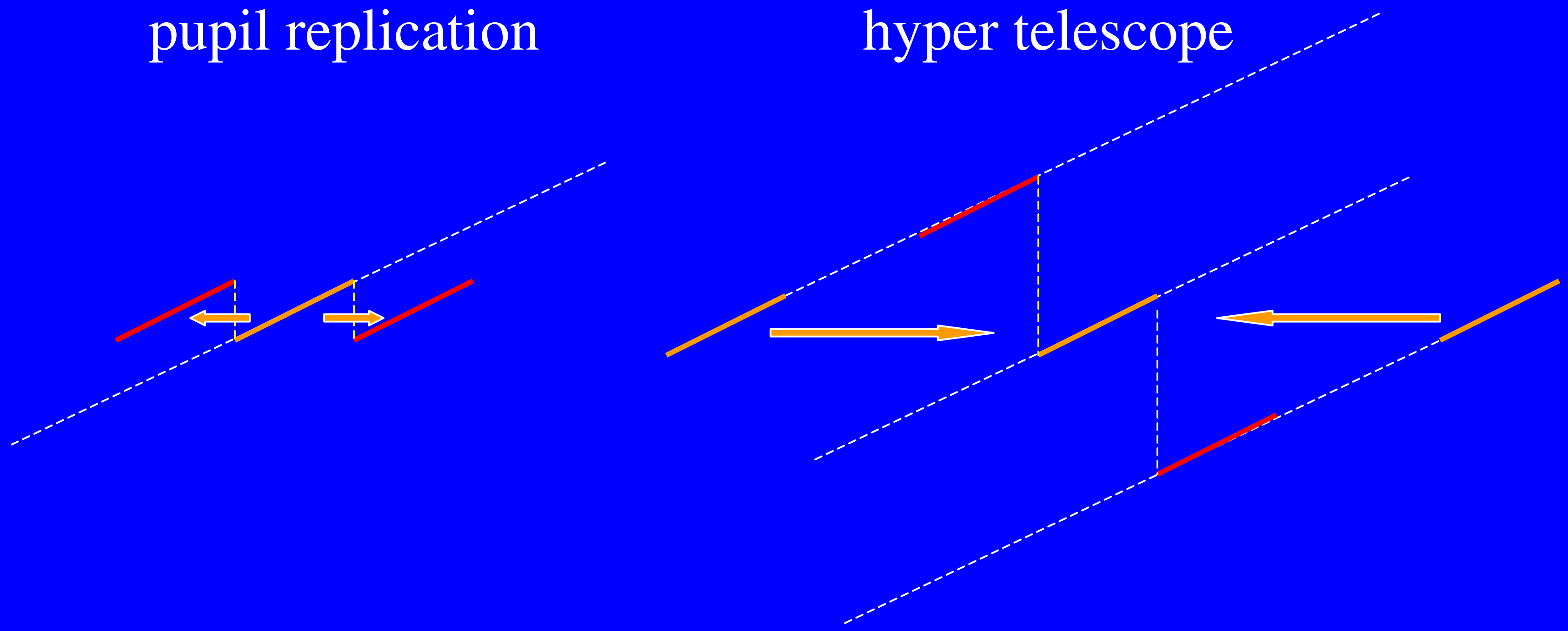
Overview

On axis: Pupil Replication = Hyper Telescope

Off axis:

pupil replication

hyper telescope



Hyper Telescope

- 1-D, 3 pupils with equal spacing h between the telescopes and joined:

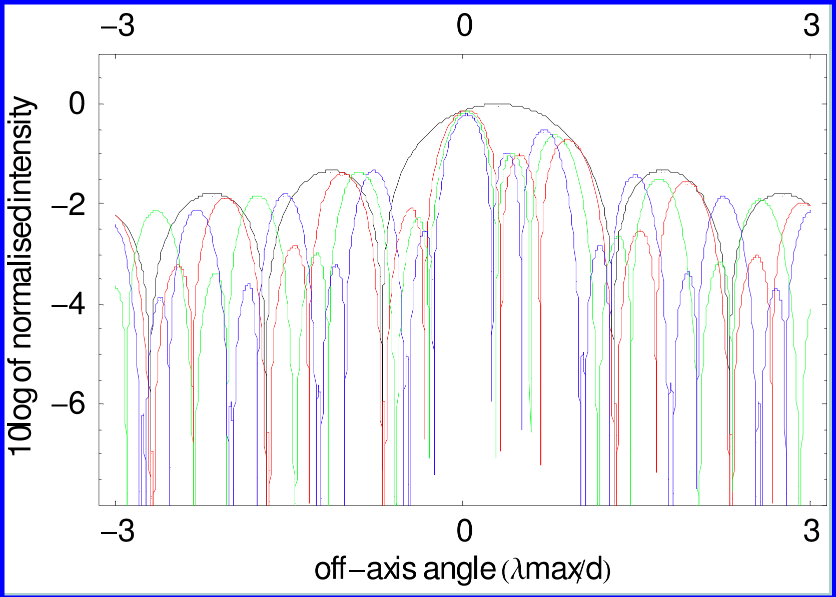
$$T = A\left(1+2\cos\left(2d\pi\left(x+((h/d)-1)\sin(\alpha)\right)/\lambda\right)\right)\operatorname{sinc}\left(d\pi\left(x-\sin(\alpha)\right)/\lambda\right)$$

  - => similar to pupil replication but:
  - => cos( ) now dependent on angle
  - on axis (star light suppression) equal to PR

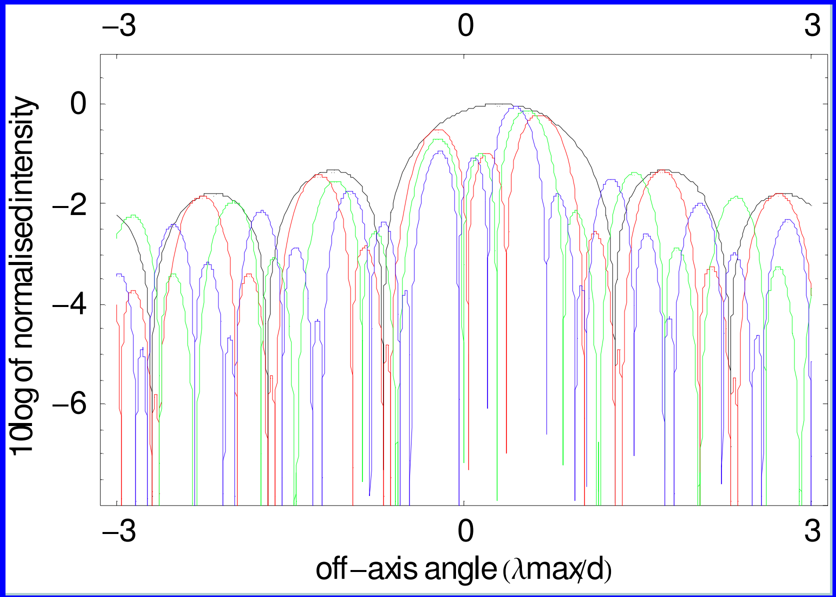
HT Analysis

- PSF: α = 0.3 λ/d of single pupil,
  h = 2d (3d centre to centre)

Pupil Replication (PR)



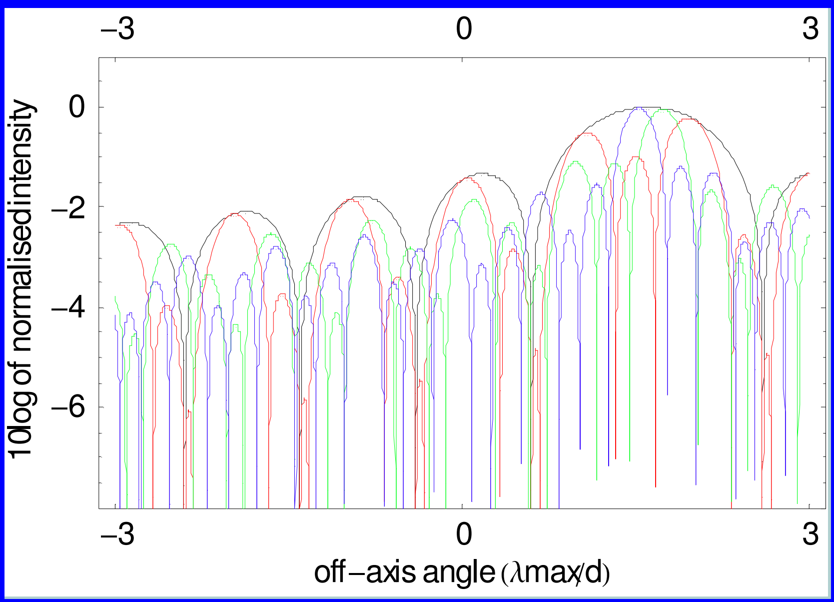
Hyper Telescope (HT)



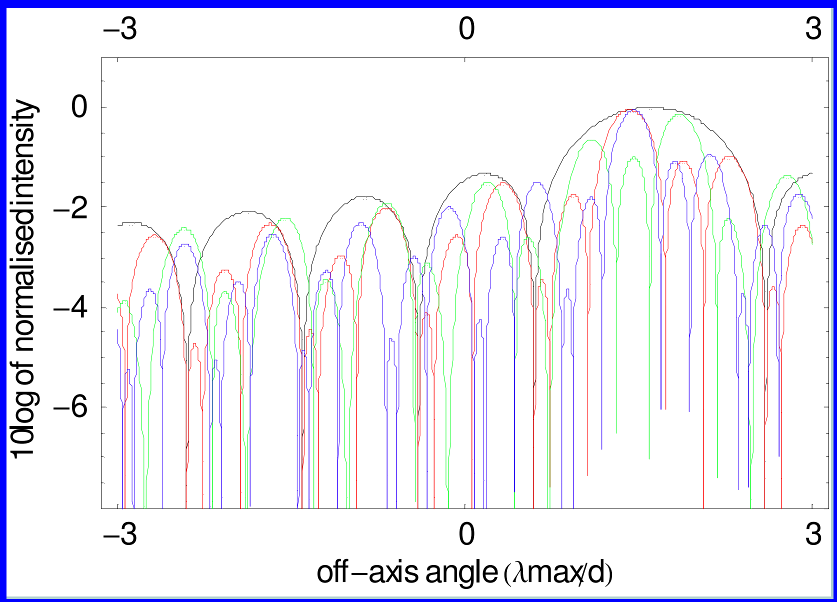
HT Analysis

- Same as last slide but:
- PSF: $\alpha = 1.6\ \lambda/d$ of single pupil,

Pupil Replication (PR)

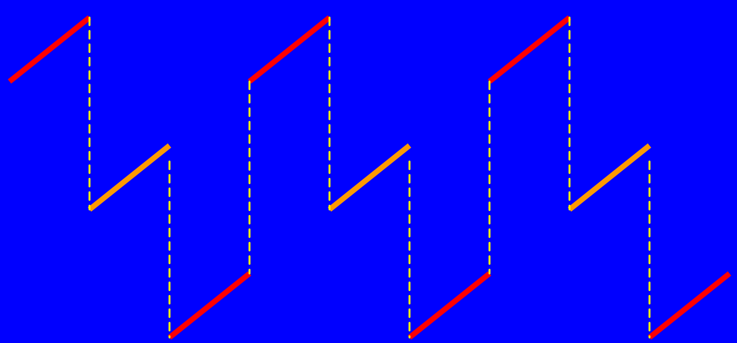


Hyper Telescope (HT)

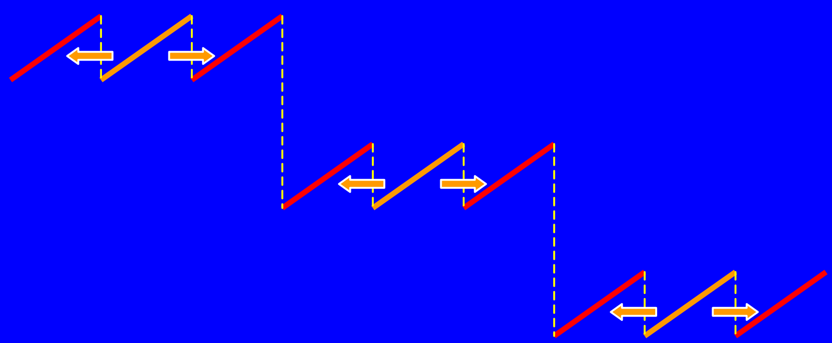


Combine?

HT-PR



PR-HT



Analysis

- HT-PR:
  the hyper telescope is replicated 3 times:

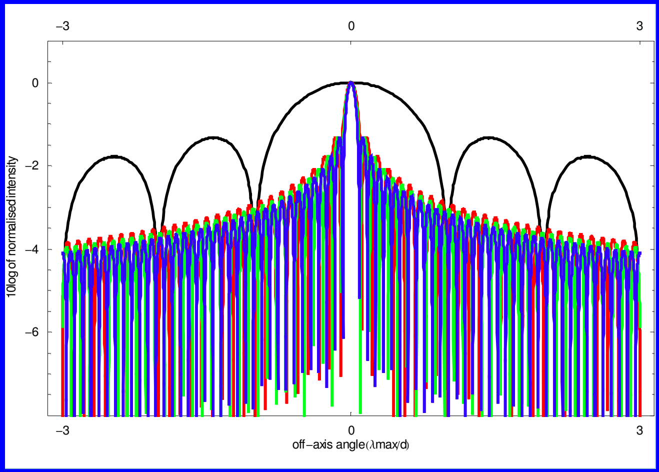
$$T_{HT-PR} = \frac{T_{HT}}{3}\left(1 + 2\cos\left(6\pi d\left(x - \sin(\alpha)\right)/\lambda\right)\right)$$

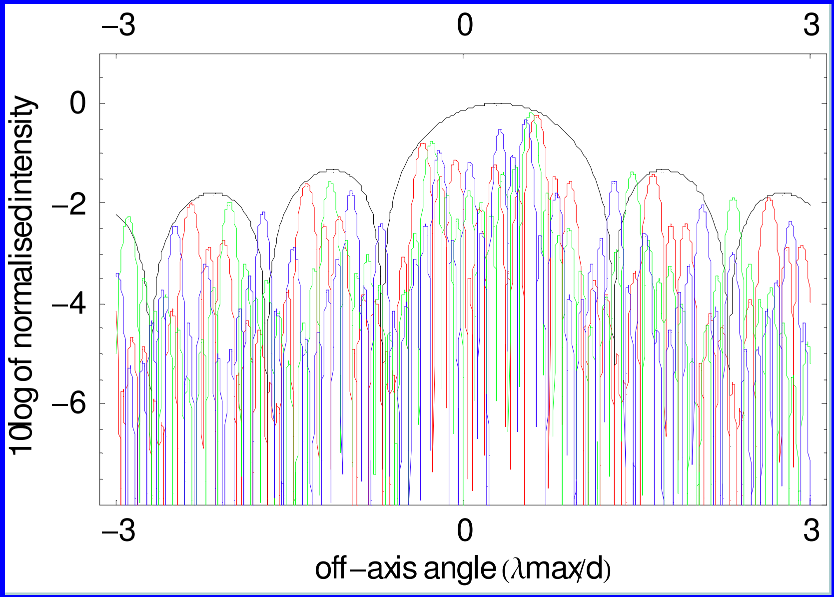
- PR-HT:
  the 3 replications are made in each of the hyper telescopes:

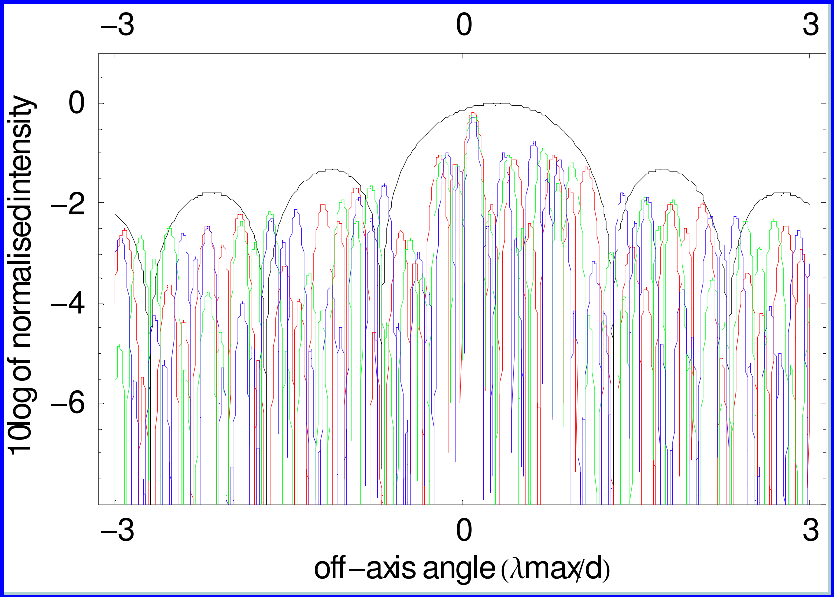
$$T_{PR-HT} = \frac{T_{HT}}{3}\left(1 + 2\cos\left(6\pi d\left(x + \left((h/d) - 1\right)\sin(\alpha)\right)/\lambda\right)\right)$$

HT-PR or PR-HT

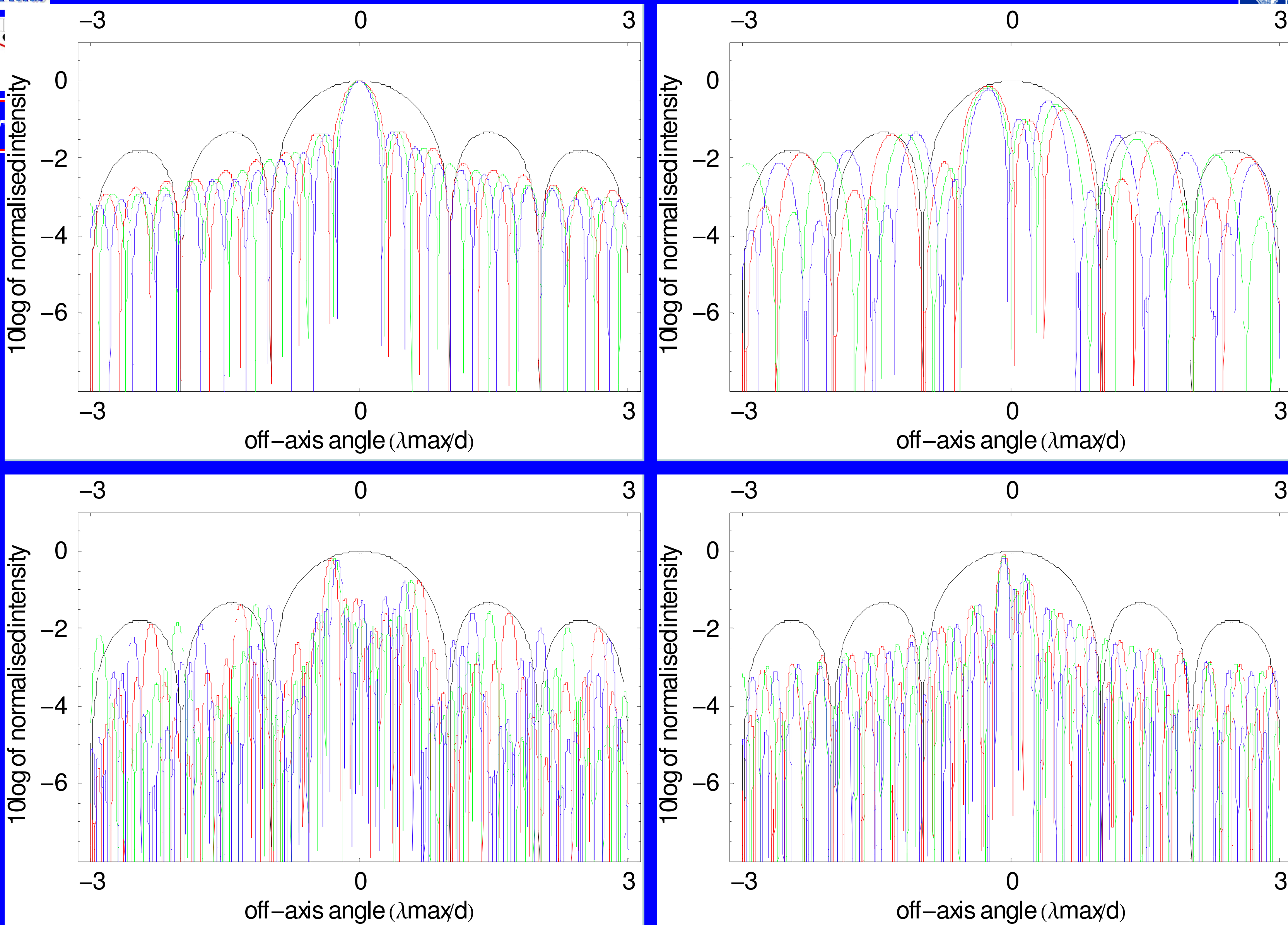
- 3x3 pupils
  $\alpha = 0.3\ \lambda/d$







H = 30 d, α = 0.01 λ/d



Evaluation?

- Based on this analysis only:
- Options:
  - pupil replication
  - hypertelescope
  - combined
  - other...
- Criteria:
  - on-axis behaviour: star suppression
  - off-axis behaviour: planet detection

Information

- Pupil Replication for Exo-Planet Imaging; A. H. Greenaway, F. H. P. Spaan and V. Mourai, The Astrophysical Journal Letters, Vol. 618-2, pp L165-L168, 10 January 2005.
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