

# Generalised Phase Diversity – Initial Tests

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## Outline

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- Numerical experiments to validate theory for generalised phase diversity.
- Description of error reduction algorithm.
- Simulation showing possible optimisation using *a priori* information
- Conclusions

## Necessary and Sufficient Conditions

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The necessary and sufficient conditions for a null wave-front sensor, which was presented in the morning by Heather Campbell, are as following

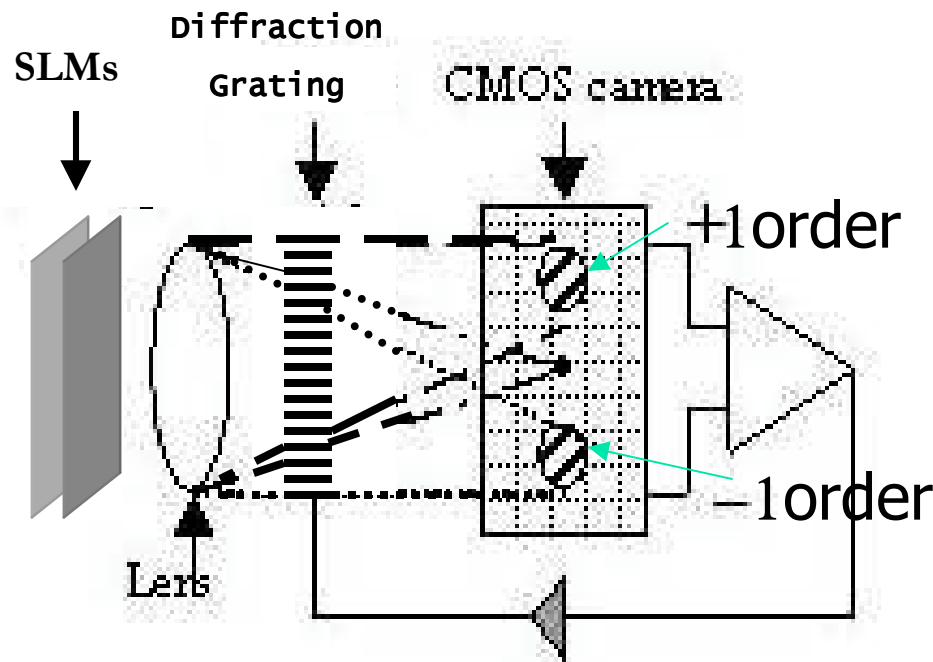
### *Sufficient Conditions:*

The difference between two aberrated diversity images is null if the input wave-front is plane wave and is non-null for non-plane wave-fronts

### *Necessary Conditions:*

The filter function must be complex.  
Mixed symmetries of the filter function must not be used

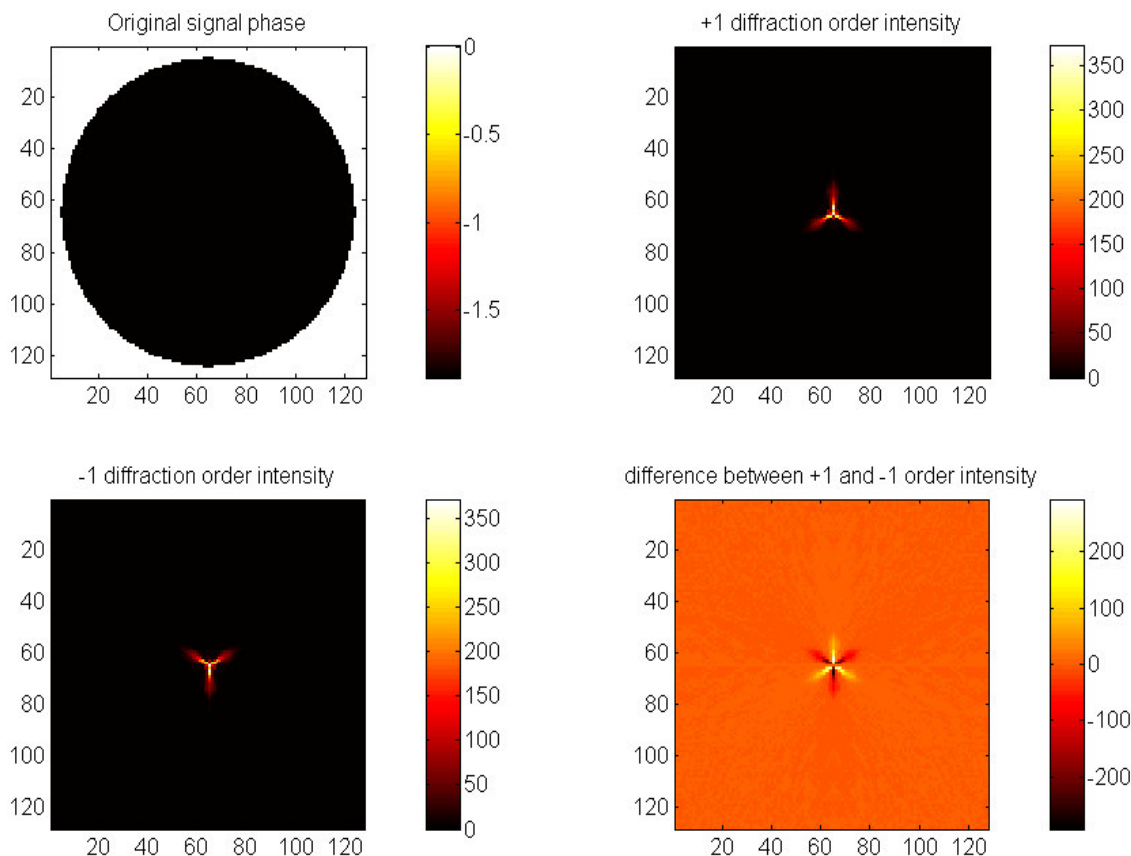
## Schematic of the wave-front sensor



- SLMs provide modulation.
- DoE provides phase-diverse data and image wave-front for correction by SLMs
- CMOS camera

A suggested Compact AO System (CAOS)

# Numerical Experiments for validating Theory



## 1 Plane wave input

$$\exp(j*1.4*\pi)$$

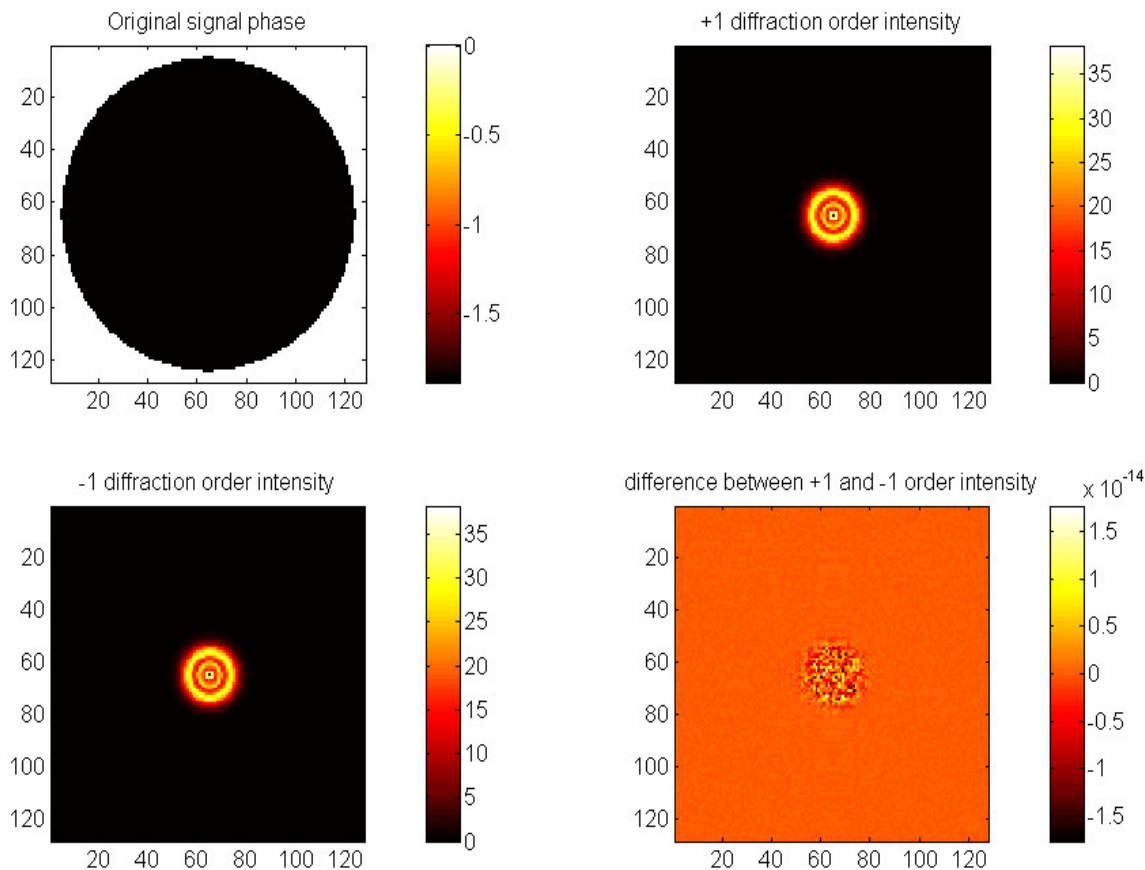
a) Mixed symmetry phase diversity:

$$z_2^0 + z_3^{-3}$$

(Defocus+Astigmatism)

Difference value for plane wave input is not zero. So mixed symmetry phase diversity is not suitable to a null sensor (Necessary Condition)

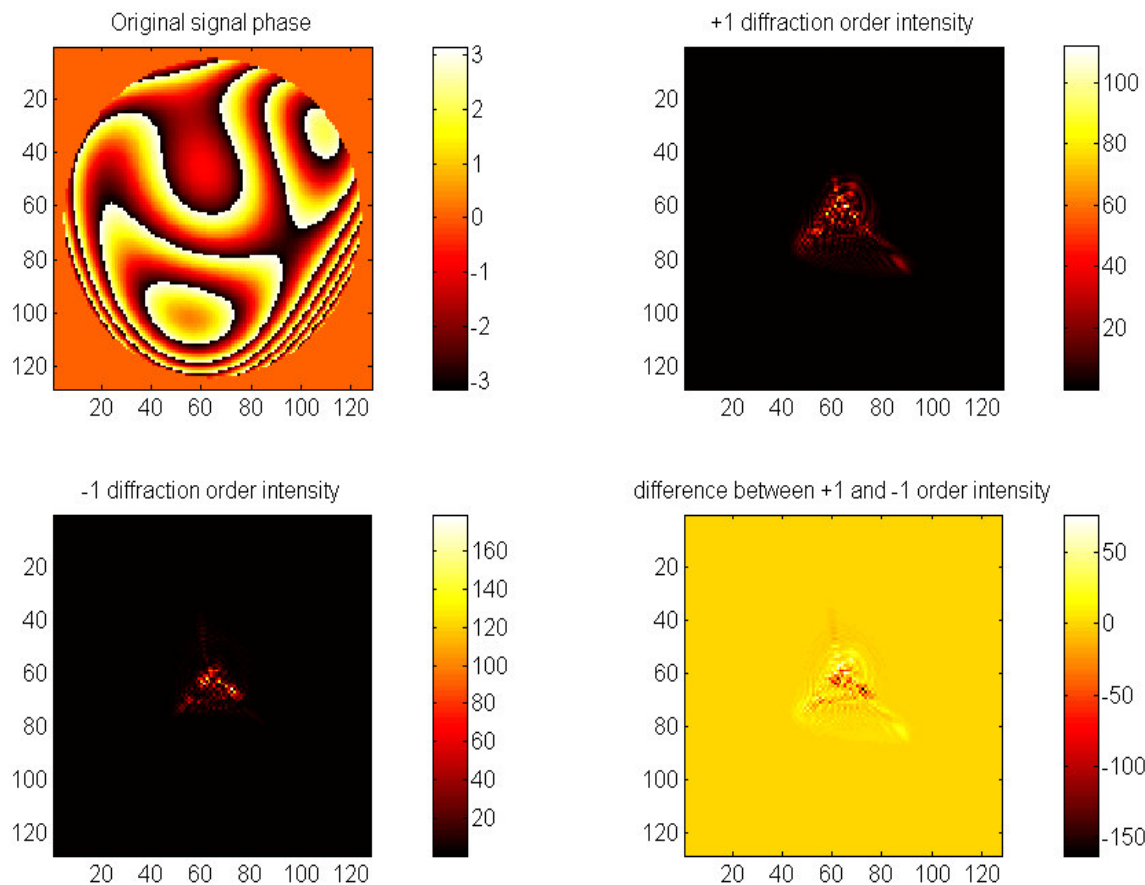
# Numerical Experiments for validating Theory



b) Even symmetry phase diversity:  
 $Z_2^0$  (Defocus)

Difference value for plane wave input is zero. So even symmetry phase diversity satisfies the sufficient condition for a null sensor

# Numerical Experiments for validating Theory

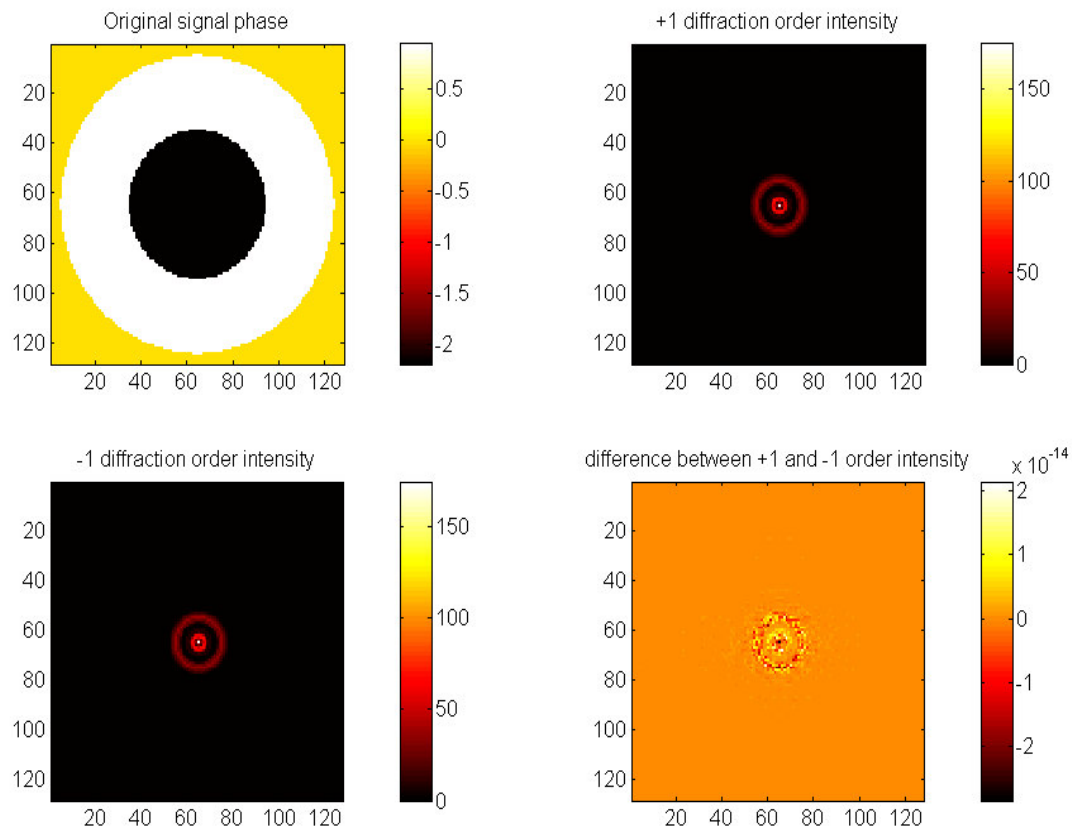


2 Non-plane wave  
input :  $\exp(j*1.4*\pi*Z)$

Even symmetry  
phase diversity:  
 $Z_{10}^0$  (Zernike)

The difference  
between two  
aberrated diversity  
images is non-null  
for non-plane wave-  
fronts(sufficient  
condition)

# Input wave-front with $\pi$ discontinuity

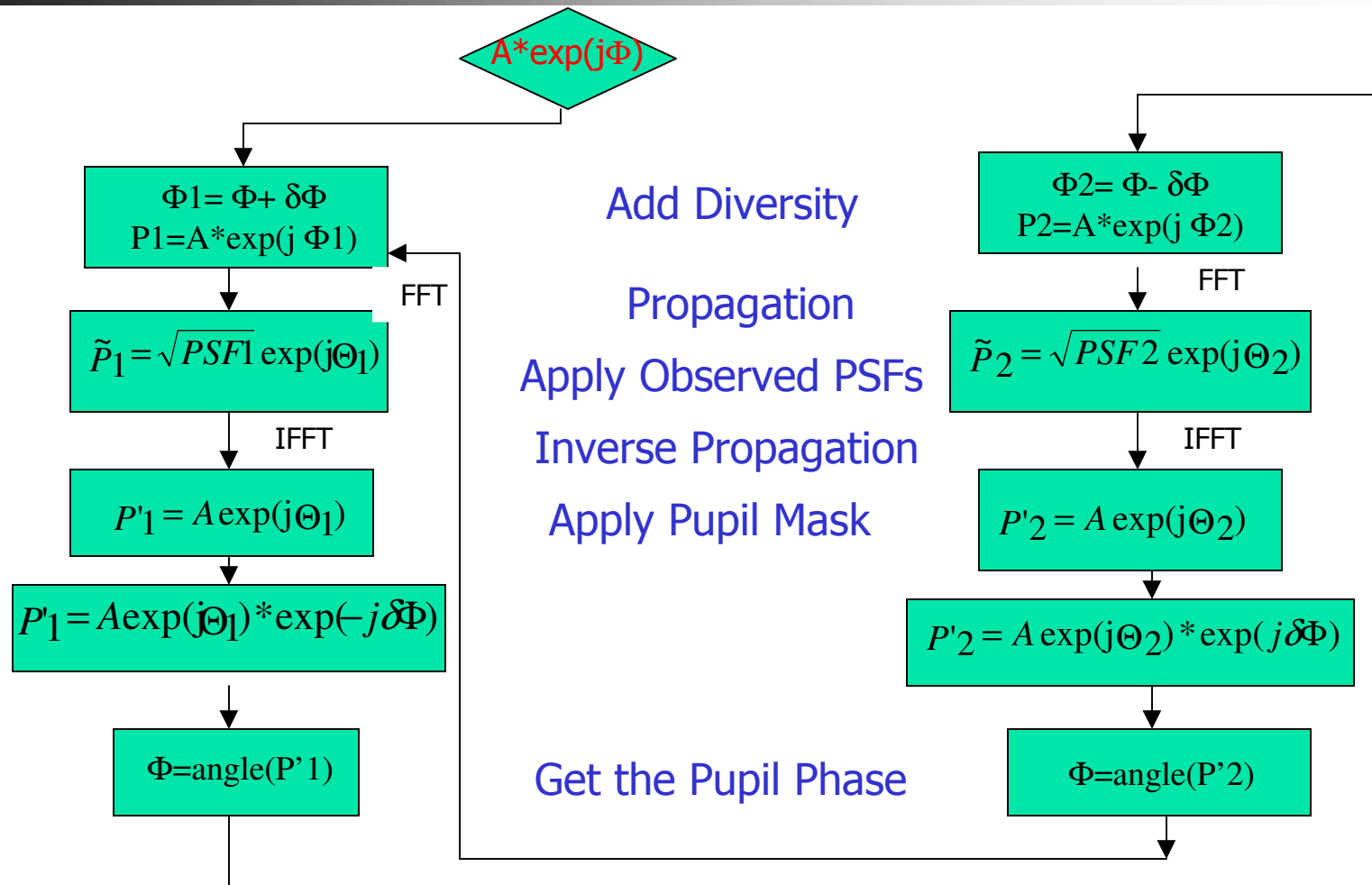


(black  $-1.3*\pi$ ; white  $-0.3*\pi$ )  
 This real wave-front is unimportant for astronomy but potentially important when using the wave-front sensor in metrology applications

Even symmetry  
 phase diversity:  $Z_2^0$

This is one of results derived from theory

# Error Reduction Algorithm for Generalised Phase Diversity



## Definition of the error metric function

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The error metric function used in simulation :

$$RMSE = [\sum(I_+ - \tilde{I}_+)^2 / (\sum(I_+^2) + \sum(I_- - \tilde{I}_-)^2 / \sum(I_-^2))] / 2$$

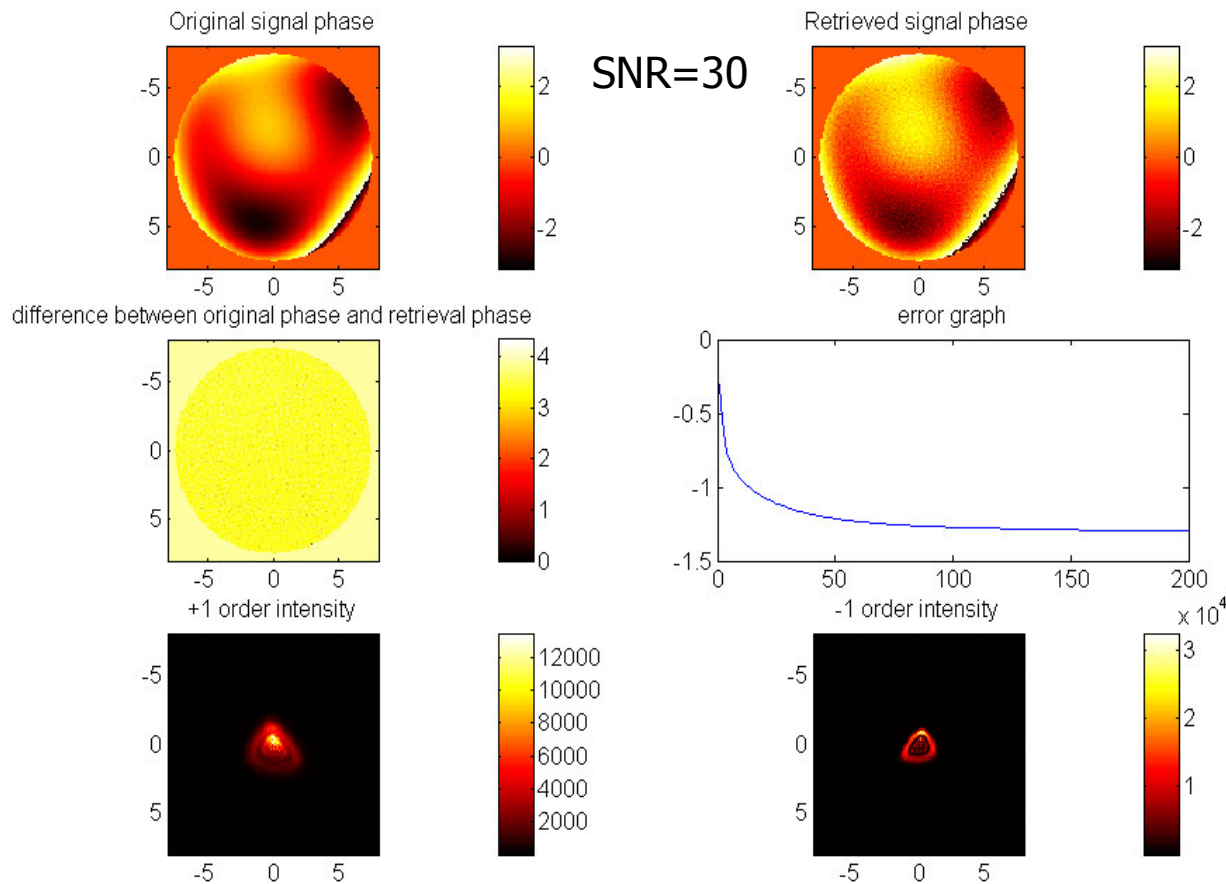
where  $I_+$  and  $I_-$  represent measured intensity data in the +1 and -1 diffraction order

$\tilde{I}_+$  and  $\tilde{I}_-$  represent an estimate of the phase diversity data intensity from the reconstructed wave-front

In simulation, we are comparing error reduction algorithm and looking for the fastest, most robust convergence properties.

Development of analytic methods for data reduction is 'in hand'

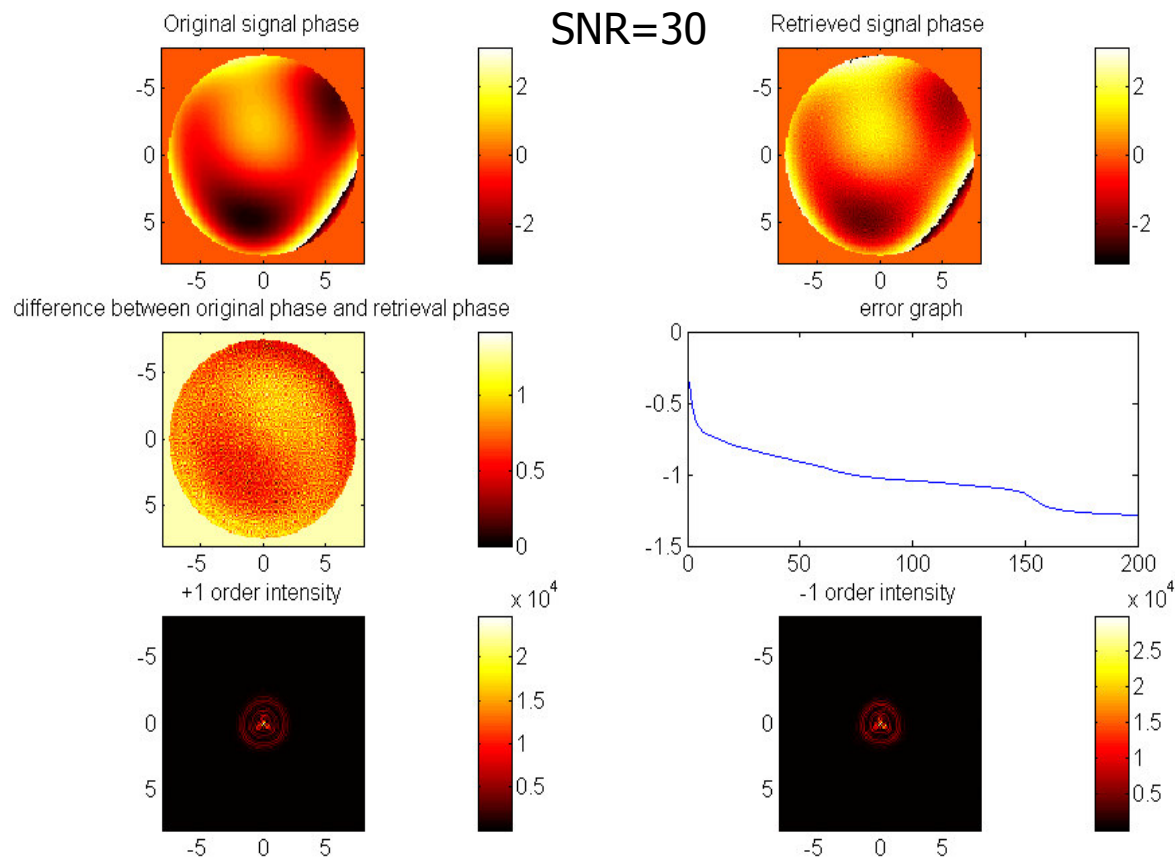
# Simulation Results for possible optimisation using a priori information



Input wave-front:  
defocus+astigmatism  
+coma+trefoil+  
spherical aberration.

Diversity filter:  $Z_2^0$   
(Defocus)

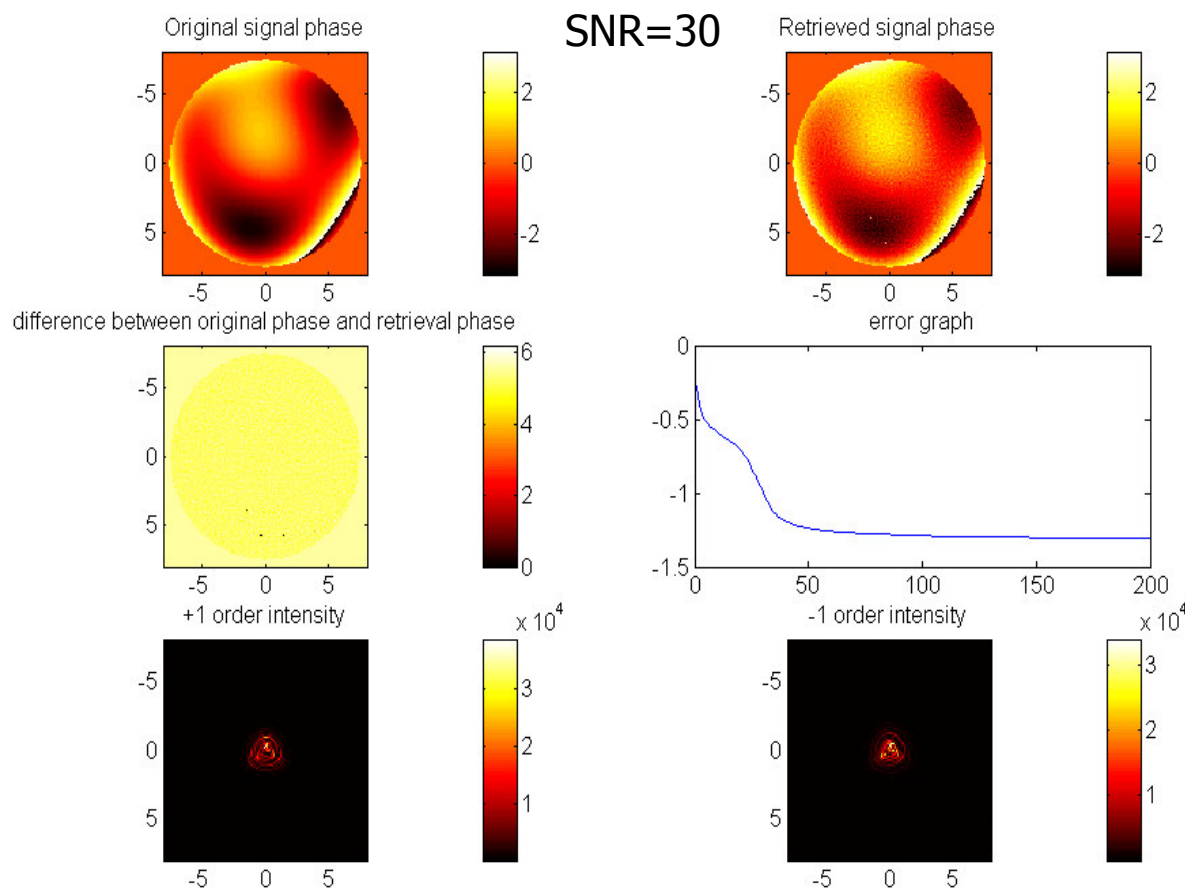
# Simulation Results(continued)



**Input wave-front:**  
 defocus+astigmatism  
 +coma+trefoil+  
 spherical aberration

**Diversity filter:**  $z_4^0$   
 (spherical aberration)

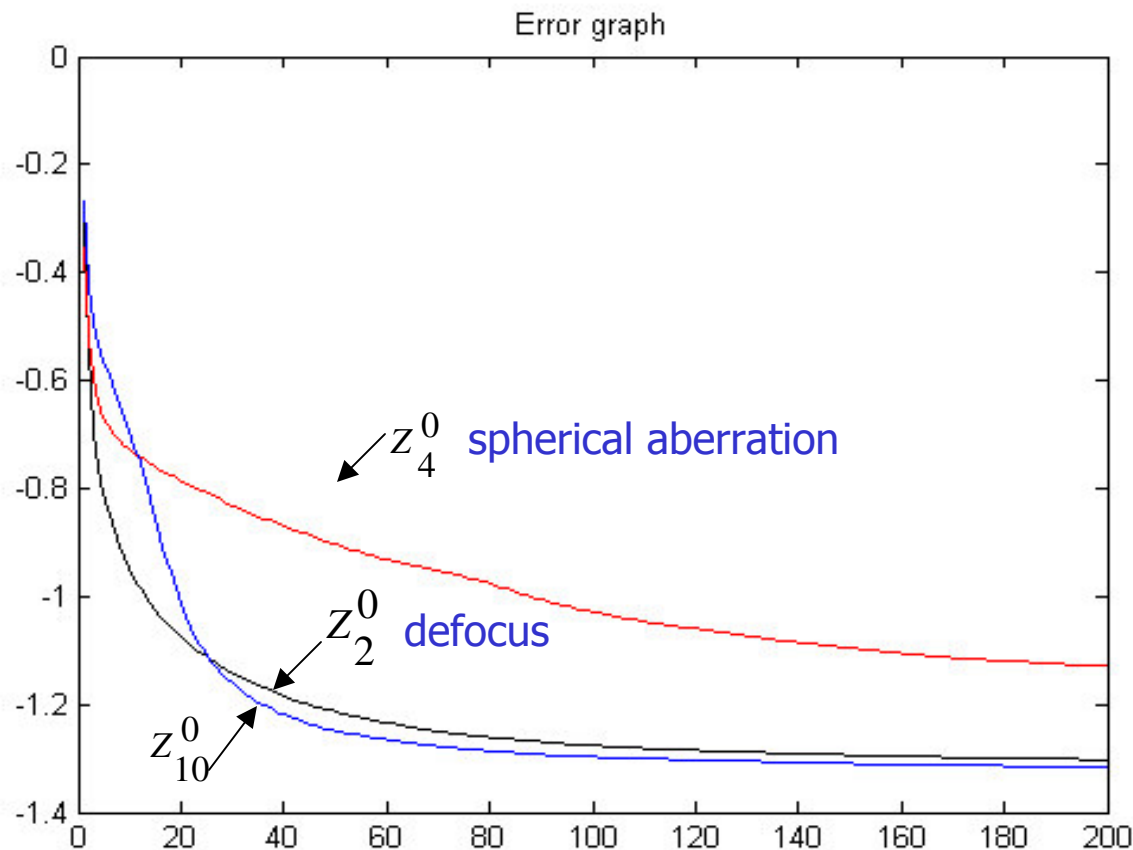
# Simulation Results(continued)



**Input wave-front:**  
 defocus+astigmatism  
 +coma+trefoil+  
 spherical aberration.

**Diversity filter:**  $Z_{10}^0$

## Comparison for three diversity cases



Shows that  $z_{10}^0$  is marginally better than defocus, although initial convergence is slower.

However, this does illustrate that different diversity kernels can be used to achieve different results

## Conclusions:

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We used numerical experiments to validate theory for the necessary and sufficient conditions for construction of a null wave-front sensor based on the principles of a generalisation of phase-diversity wave-front sensing.

So far we find no deviations numerical experiment results from theoretical results.

Describing generalised error reduction algorithm.

Simulation results illustrated that the generalised wave-front sensor can be optimised to exploit *a priori* information in order to maximise sensitivity of wave-front sensor.