Small angle expansion, a solution to the phase-retrieval problem using generalised phase diversity in pupil space

Sijiong Zhang, Heather I. Campbell, and Alan H. Greenaway

Physics, Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, Scotland, EH14 4AS

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Overview Of Presentation.

- Defocus Phase Diversity(DPD) wavefront sensing
- Generalised Phase Diversity (GPD) wavefront sensing
- Implementation of Generalised Phase Diversity
- Phase diverse data expression for GPD
- Small Angle Expansion (SAE).
- Simulation Validation for SAE
- Conclusions

Defocus Phase Diversity

- Defocus phase diversity is a phase-retrieval algorithm that uses a pair of intensity images taken symmetrically about the wave-front to be determined

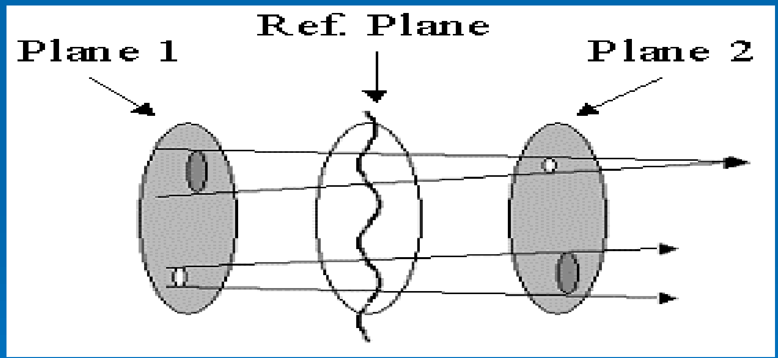


Fig.1 Relationship between two defocus intensities and wavefront curvature

Defocus Phase Diversity(Cont.)

- Wavefront curvature is related to axial intensity derivative through the Intensity Transport Equation(ITE).

$$-\frac{k}{I}\frac{\partial I(s)}{\partial z} = \nabla^2 \phi(s)$$

- where k is wave number, I is intensity, z is optical axis, and ϕ is the phase.

- limitations of ITE: It requires the continuity of the complex amplitude and its derivative which will preclude some interesting applications.

Generalised Phase Diversity

- The defocus phase diversity algorithm provides accurate, robust and real-time solutions to the phase problem in optics.
- How to preserve these favourable properties whilst relaxing the imposition of *a priori* requirements on the wavefront to be reconstructed?
- Solution--Generalised Phase Diversity(GPD)

Generalised Phase Diversity(Cont.)

- Generalised phase diversity should include defocus phase diversity.

- The filter function used for GPD satisfies the necessary and sufficient conditions:

  a) a null output for plane wavefronts,
  and an error signal for distorted wavefronts;

  b) filter function with ‘same symmetry’.

Generalised Phase Diversity(Cont.)

Physical description of GPD

- The error signal for GPD can be thought of as a convolution of the input wave function with a defined 'blur function' which is related to the phase diversity filter.
- the convolution integral effectively sums the input wavefront, weighted by the blur function, over the area where the blur function has significantly non-zero values.
- The weighted contributions from different parts of the wavefront thus interfere.

Implementation of GPD

- One practical implementation of GPD using a diffractive optical element in pupil space

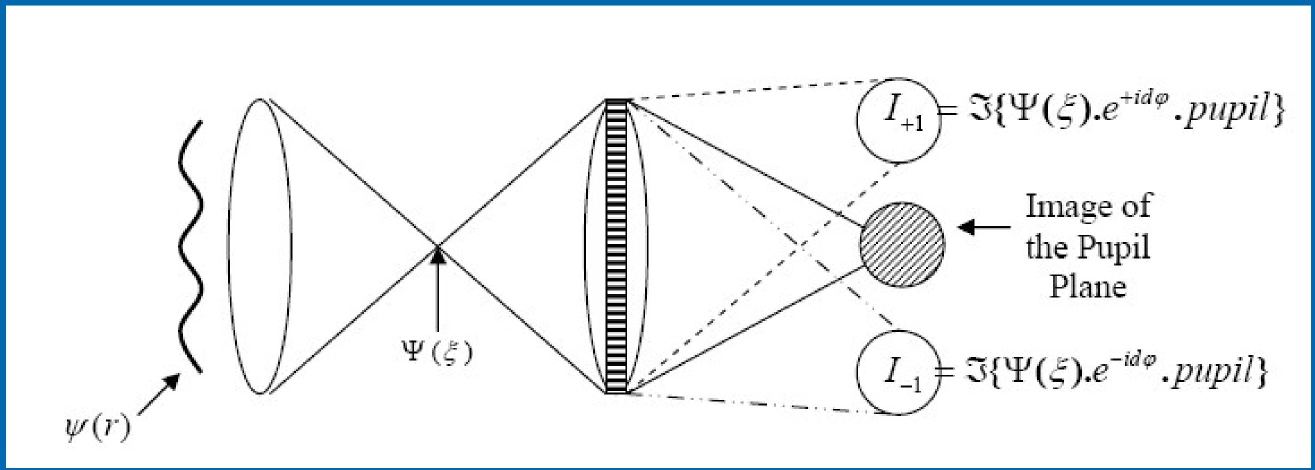


Fig.2 Schematic of generalised phase diversity with DOE from the paper by Campbell (see this conference)

Phase diverse data expression for GPD

$$\frac{d(r)}{2i} =$$

$$\Big[\int d\xi\, H(\xi) I(\xi) \exp(-ir\xi) \int d\xi'\, A^*(\xi') R(\xi') \exp(ir\xi') -$$

$$\int d\xi\, A(\xi) R(\xi) \exp(-ir\xi) \int d\xi'\, H^*(\xi') I(\xi') \exp(ir\xi')\Big] +$$

$$\Big[\int d\xi\, A(\xi) I(\xi) \exp(-ir\xi) \int d\xi'\, H^*(\xi') R(\xi') \exp(ir\xi') -$$

$$\int d\xi\, H(\xi) R(\xi) \exp(-ir\xi) \int d\xi'\, A^*(\xi') I(\xi') \exp(ir\xi')\Big]$$

Small Angle Expansion (SAE)

- we wish to use a diversity function other than just defocus function, but the above phase diverse data expression is not in a convenient form for phase solution and the intensity transport equation is no longer valid.

- We need to look for a new algorithm for solving above phase diverse data expression for phase.

- An approach to the problem---small angle expansion, which leads to a very simple analytic formula for phase.

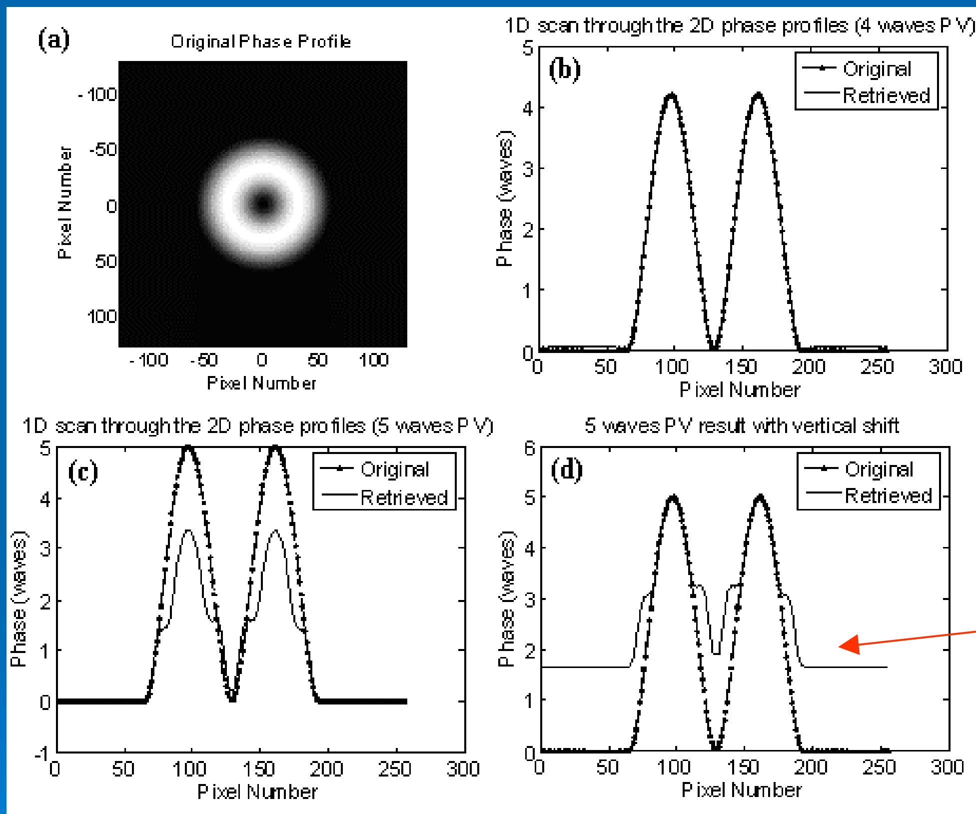
Small Angle Expansion (Cont.)

Validity of SAE

- the SAE approximation is valid over the region of the blur function if the phase change is less than $\lambda/4$.
- the small-angle approximation limits the rate of change of the wavefront phase that can be reconstructed rather than the overall peak-to-valley of the phase distribution of the wavefront.

Simulation Validation for SAE

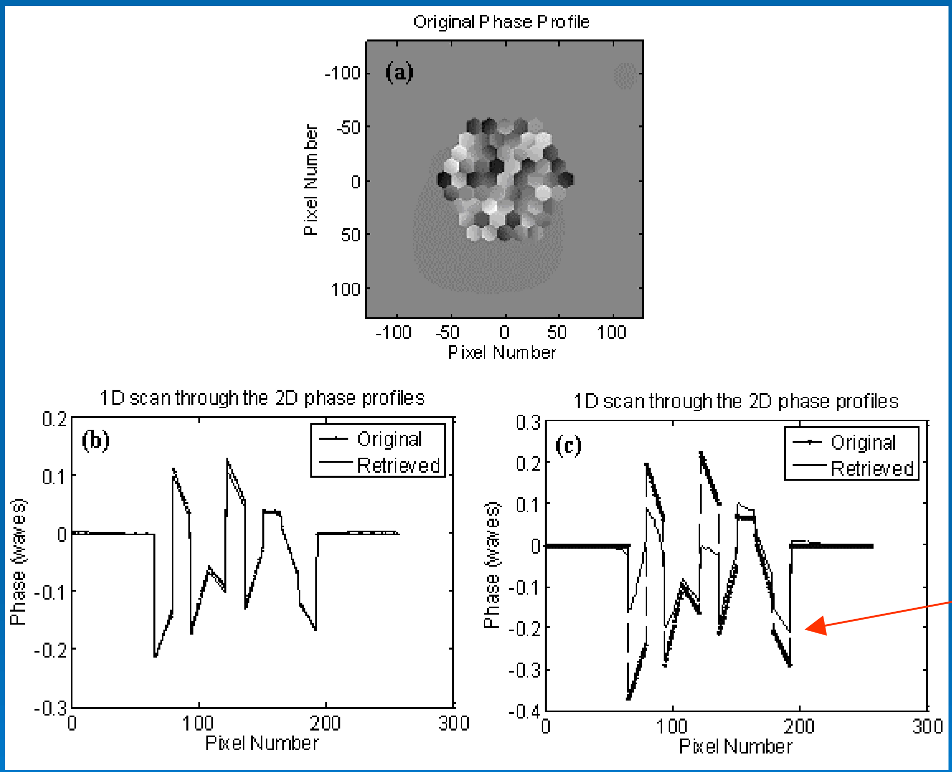
➢ Non-scintillated wavefront



the wavefront phase is too steep in middle position

Fig.3 a continuous wavefront(soft edge)

Simulation Validation for SAE(Cont.)

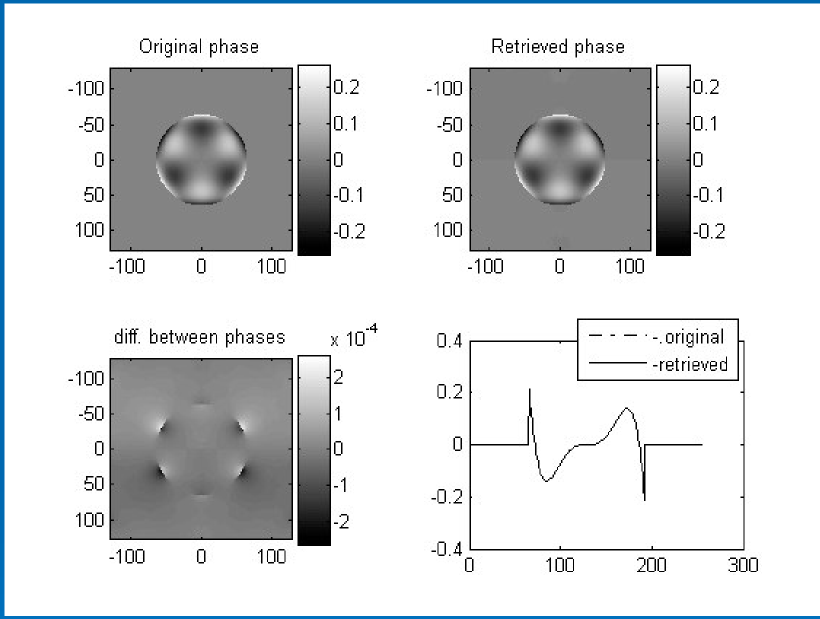


The phase change exceeds $\lambda/4$

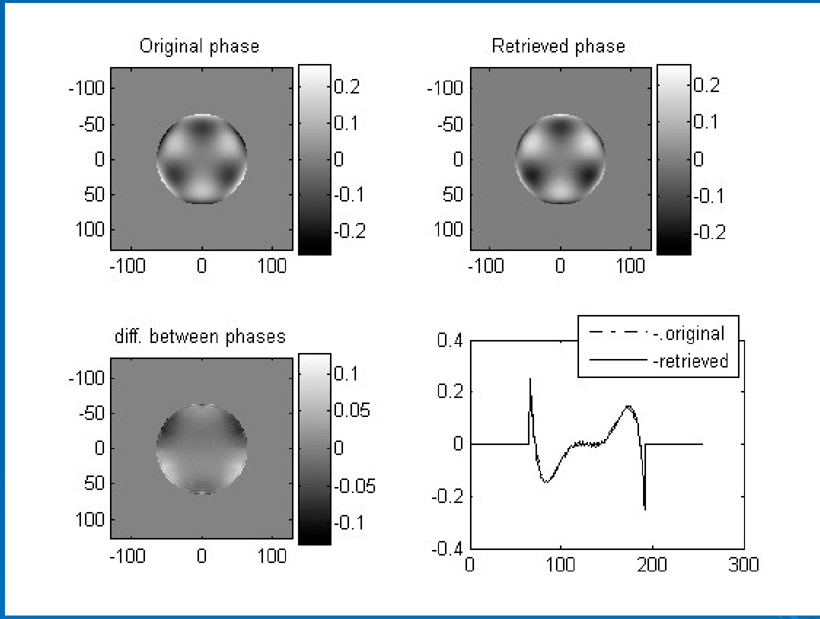
Fig.4 a discontinuous wavefront(soft edge)

Simulation Validation for SAE(Cont.)

- Boundary problem



Soft edge

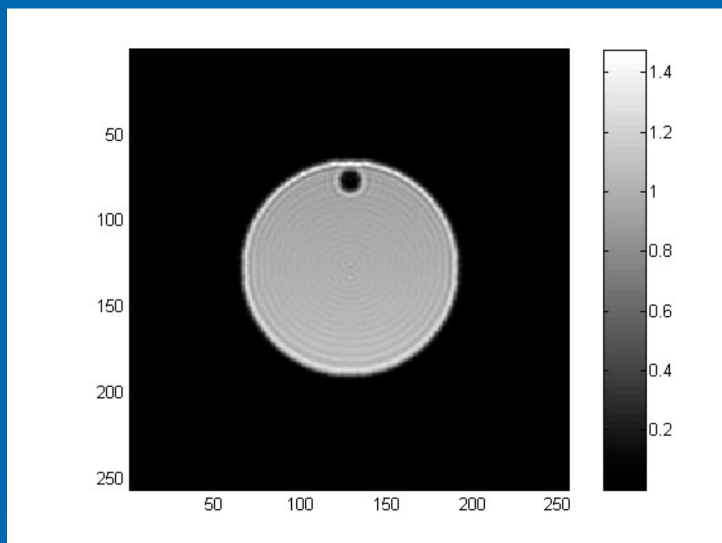


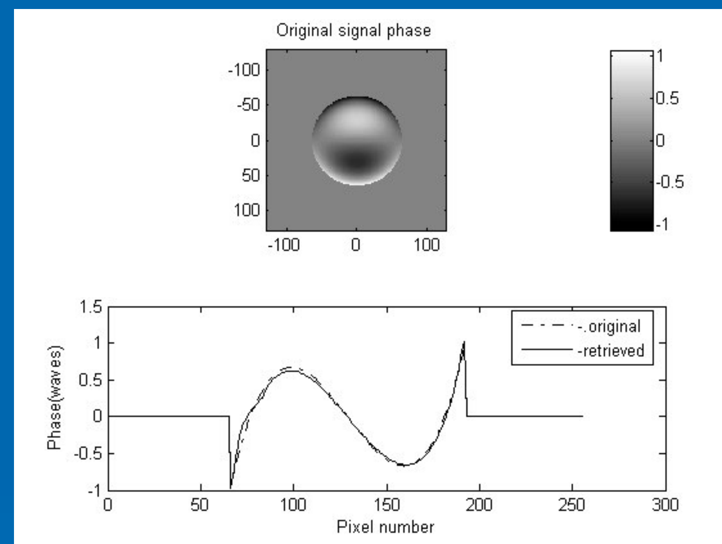
Hard edge

Fig.5 a continuous wavefront

Simulation Validation for SAE(Cont.)

- Effect of obscuration wavefronts(hard edge)





(a) intensity distribution; (b) phase distribution

Fig.6 a moderately-scintillated wavefront.

(The cut is taken vertically through the spot which can be noticed on the left hand side of the plot)

Boundary problem

- We note here that we have found that the treatment of the boundary-value data is crucial to obtaining a good wavefront reconstruction where the test wavefront is modeled as having a hard edge—such as a mirror edge or secondary mirror structure.

- In general we have found that where input wavefront is modeled as soft edge (for example a TEM_{00} mode) the reconstruction obtained are superior. Figure 5 shows the situation.

Future Work.

- Find a best approach for dealing with boundary-value problem.

- Wavefront reconstruction with strong scintillation.

Conclusions.

- We developed a phase reconstruction method for GPD wavefront sensors that gives good phase reconstruction in situations that are incompatible with the underlying mathematical assumptions in ITE;
- the approach SAE presented here is capable of providing accurate reconstruction for wavefronts that are discontinuous and/or are subject to modest levels of scintillation;
- the SAE algorithm offers significant computational savings

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